

## Split Personality

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**Carbonates and sandstones differ in a number of fundamental ways** (Gene Ballay. 2005), with consequences that affect the techniques required for their evaluation (Chris Smart, 2003). One outcome of these differences is **the likelihood of a multi-mode porosity system in carbonates**, which in a manner akin to that thriller *Dr Jekyll and Mr Hyde*, can consist of pores that are almost art from a visual perspective, but become sinister when one is charged with correctly evaluating the reservoir.

In a recent Abu Dhabi Topical Conference (Chris Smart. 2005), the three most common causes of carbonate low resistivity pay were identified as (ranked from most common, downwards).

1. Dual or even triple porosity systems, interspersed amongst one another.
2. Layered reservoirs, with the layers consisting of different pore sizes.
3. Fractured reservoirs.

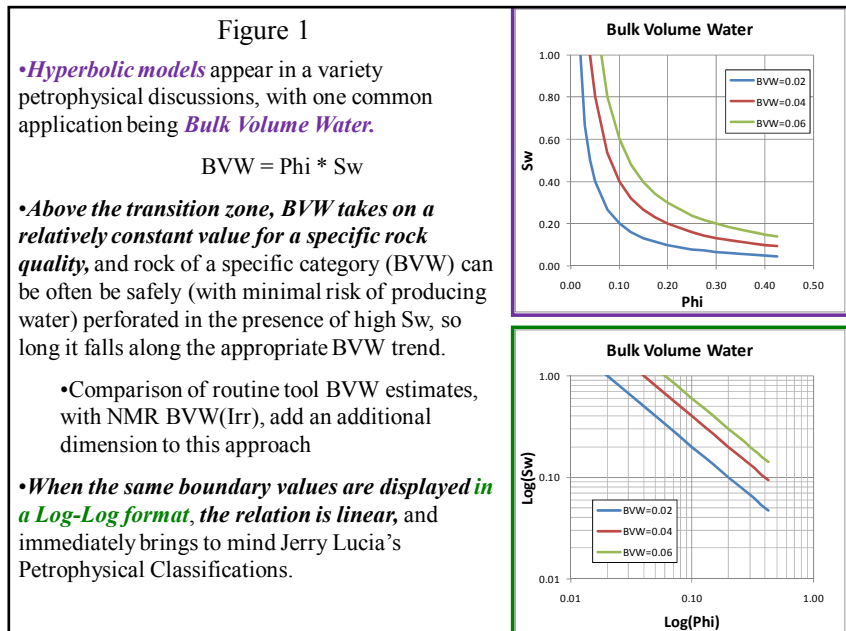
In all three cases, **the fundamental issue is one multiple pore systems.**

The variable size pores may be visually evident in the rock, or they may manifest their presence only in capillary pressure curves. In either situation, **it is often with mercury injection capillary pressure data** (Bob Purcell, 1949 and 1950) **that one will begin to quantify the issue, and we are then in immediate need of a physically meaningful mathematical framework within which to perform that quantification.**

## Hyperbolic Models

**Hyperbolic models appear in a variety petrophysical discussions**, with one common application being **Bulk Volume Water**:  $BVW = \Phi * S_w$ . Above the transition zone, BVW takes on a relatively constant value for a specific rock quality, and rock of a specific category (BVW) can be often be safely (with minimal risk of producing water) perforated in the presence of high  $S_w$ , so long it falls along the appropriate BVW trend (Ross Crain, 2009).

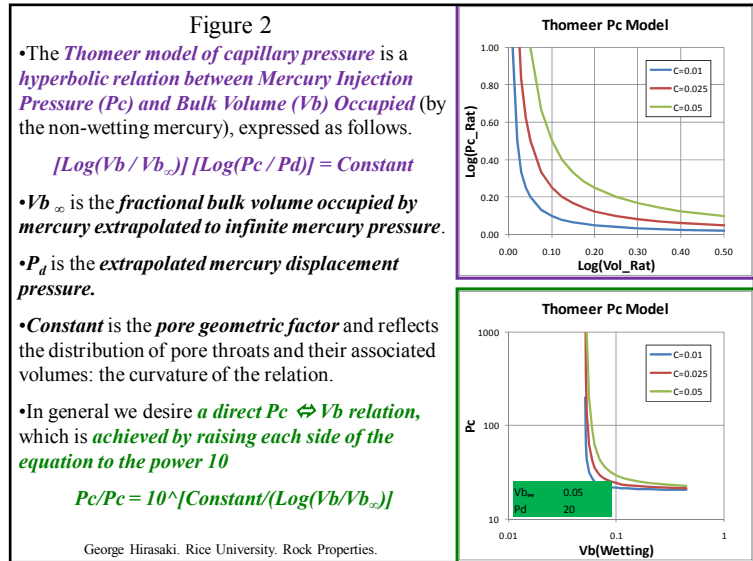
As a specific example, the Kansas Geological Survey summarizes the following generic BVW(Critical) values.



- BVW(Vuggy Carbonate) ~ 0.02.
- BVW(IX/IG Carbonate) ~ 0.04.
- BVW(Sandstone) ~ 0.06.

**In a linear format, the trends are,** as the characterization implies, **hyperbolic**. The same trends, on a Log-Log display, become linear, and immediately bring to mind Jerry Lucia's Petrophysical Classifications: Figure 1.

The value of the BVW constant, from one trend to the next, is such as to alter the placement and curvature of the constraint.



### The Thomeer Model

The **Thomeer model of capillary pressure is a hyperbolic relation between Mercury Injection Pressure (Pc) and Bulk Volume (Vb) Occupied** (by the non-wetting mercury), expressed as follows.

$$[\text{Log}(Vb / Vb_{\infty})] [\text{Log}(Pc / Pd)] = \text{Constant}$$

- $Vb_{\infty}$  is the **fractional bulk volume occupied by mercury extrapolated to infinite mercury pressure**: the vertical asymptote.
- $P_d$  is the **extrapolated mercury displacement pressure** in psi: the pressure required to enter the largest pore throat: the horizontal asymptote.
- **Constant** is the **pore geometry factor**, the distribution of pore throats and their associated volumes: the curvature of the relation.

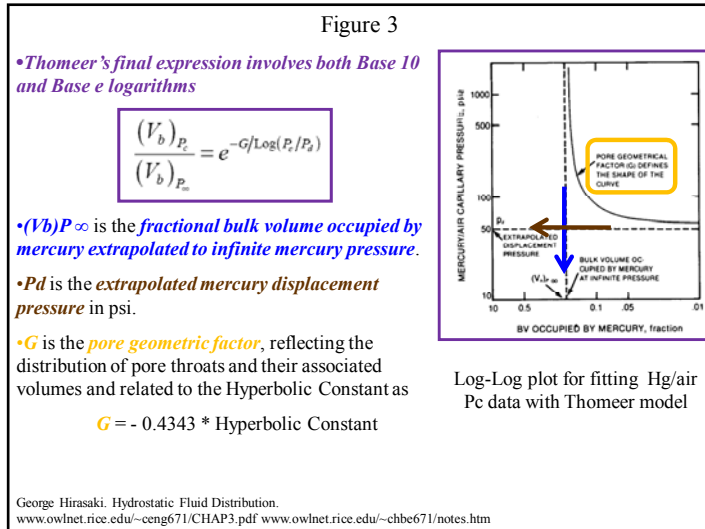
$Vb_{\infty}$  is about equal to the sample porosity for high permeability rock, but can be different in lower quality rock.

The formulation is sufficiently general that  $P_d$  may vary by a power of ten, while Constant remains nearly unchanged (ie the size of the grains spans a range of values but the curvature of the Pc curve remains similar): Figure 2.

In practice, upon application we typically express one variable as a function of the other (rather than the product being a constant), and so the relation is written as below.

$$[\text{Log}(Vb / Vb_{\infty})] = \text{Constant} / [\text{Log}(Pc / Pd)]$$

One proceeds to a direct (non-logarithmic) expression for  $Vb / Vb_{\infty}$  by raising each side of the equation to the power 10, Figure 3 per George Hirasaki, and then introducing the Natural Logarithm/Exponential.



$$V_b / V_{b\infty} = 10^{\{\text{Constant}/[\text{Log}(P_c / P_d)]\}}$$

See discussion on logarithms in Appendix

$$V_b / V_{b\infty} = 10^{\{\text{Constant} / [\text{Log}(P_c / P_d)]\}} \rightarrow \exp[-G/\text{Log}(P_c / P_d)]$$

$$-G = 0.4343 * \text{Constant}$$

Note that both common and natural logarithms are being referenced, Base 10 and Base e. When drawing upon someone else's curve fit parameters, or performing our own, we must follow a consistent use of the two logarithmic bases.

The hyperbolic approach is in fact of general utility, and **could be potentially used** (for example) **to describe the Saturation – Height relation** (Craig Phillips, 2009).

$$[\text{Log}(S_w / S_{wirr})] [\text{Log}(\text{Height} / \text{FreeWaterLevel})] = \text{Constant}$$

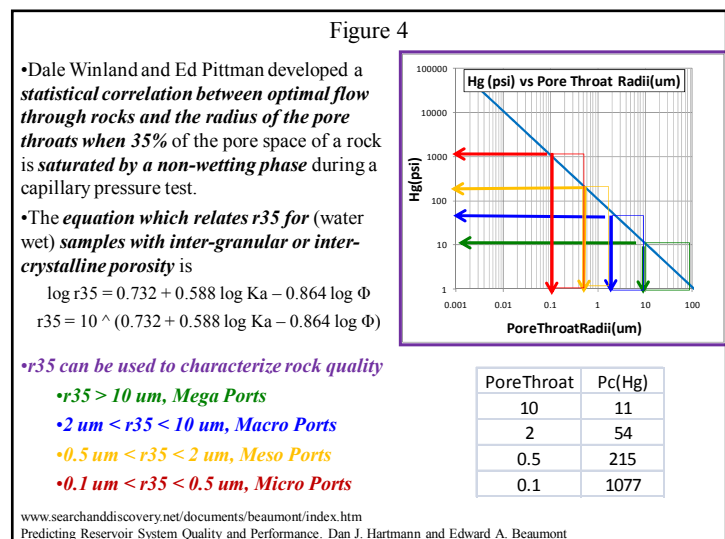
**Once the basic concept is understood, there are multiple applications.**

### Capillary Pressure Curve Attributes and Rock Quality

In addition to specifying the Saturation – Pressure – Height relation, capillary pressure curves provide a direct indication of rock quality, one sample to the next. The Lucia System, for example, is formulated in a manner which allow visual implementation (in the field or core shed), but has as its basis an observed relationship between capillary displacement pressure and grain / crystal size.

On the other hand, we sometimes find ourselves doing a field study years after the wells were drilled / cored, and with very little rock to actually examine (and classify). In this situation a classification scheme based directly upon the Pc curves becomes attractive: Figure 4.

Dale Winland and Ed Pittman examined correlations of porosity, permeability and capillary pressure curves to recognize an optimal relation against r35, the pore throat radius being touched by the non-wetting mercury at 35% saturation.



As discussed in detail by Hartmann,  $r_{35}$  breaks the Phi-Perm crossplot into domains similar to (the perhaps more common) Permeability/Porosity ratio, but has the attraction of being a physically meaningful attribute; the pore throat radius being touched when the non-wetting phase saturation is 35%.

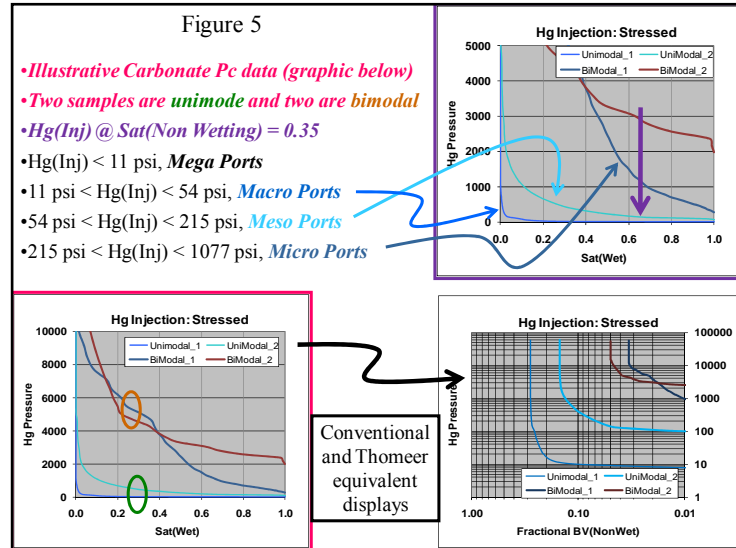
$r_{35}$  is directly related to the corresponding mercury injection pressure, and can be used as a generic rock quality indicator.

$r_{35} > 10 \text{ } \mu\text{m}$ , **Mega Ports**

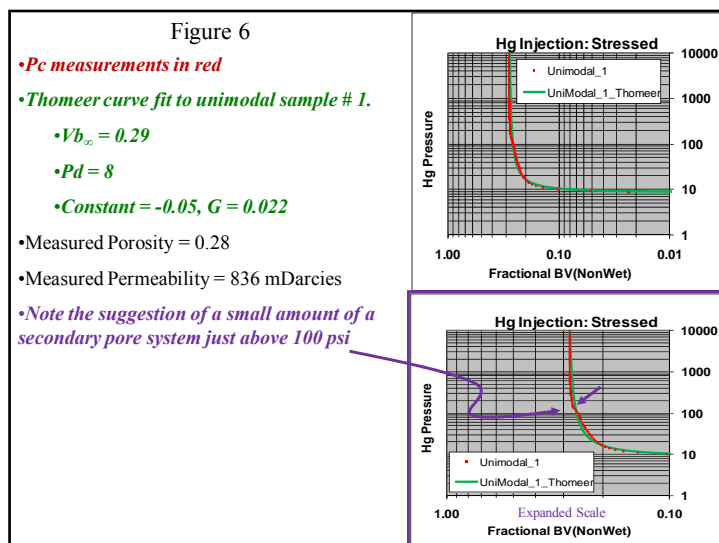
$2 \text{ } \mu\text{m} < r_{35} < 10 \text{ } \mu\text{m}$ , **Macro Ports**

$0.5 \text{ } \mu\text{m} < r_{35} < 2 \text{ } \mu\text{m}$ , **Meso Ports**

$0.1 \text{ } \mu\text{m} < r_{35} < 0.5 \text{ } \mu\text{m}$ , **Micro Ports**



### Carbonate Capillary Pressure Curves



At the simplest level, carbonate Pc curves represent a single set of pore body / throat sizes, and are thus amenable to standardized interpretation.

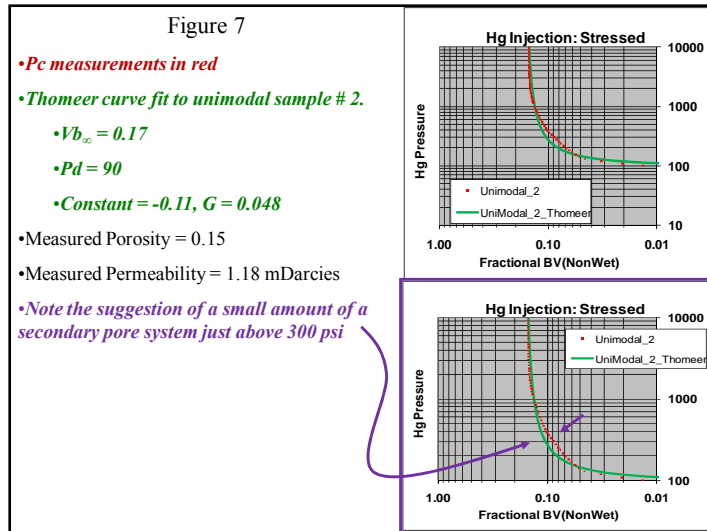
Even in this simple case, however, the Thomeer formulation deserves consideration because of the mathematical versatility of the formulation, the shareware Excel curve fitting software which Ed Clerke distributes and the potential to cluster Thomeer attributes for Rock Type identification (Clerke, 2004).

In practice, the Pc curves may be a combination of uni- and bi-mode responses, and perhaps even more complex than that: Figure 5.

It's also worth pointing out that, particularly in the case of legacy data, the measurements may not have been made at reservoir conditions (Mitchell, 2003) and one should be alert for the implications.

Figure 6 illustrates the Thomeer curve fitting procedure in the case of a uni-modal sample.

$$Vb / Vb_{\infty} = 10^{\{ \text{Constant} / [\text{Log}(Pc / Pd)] \}} = \exp[-G / \text{Log}(Pc / Pd)]$$



A single Thomeer hyperbola reasonably represents the data, with  $Vb_{\infty} = 0.29$  comparing favorably to the measured porosity of 28 pu.

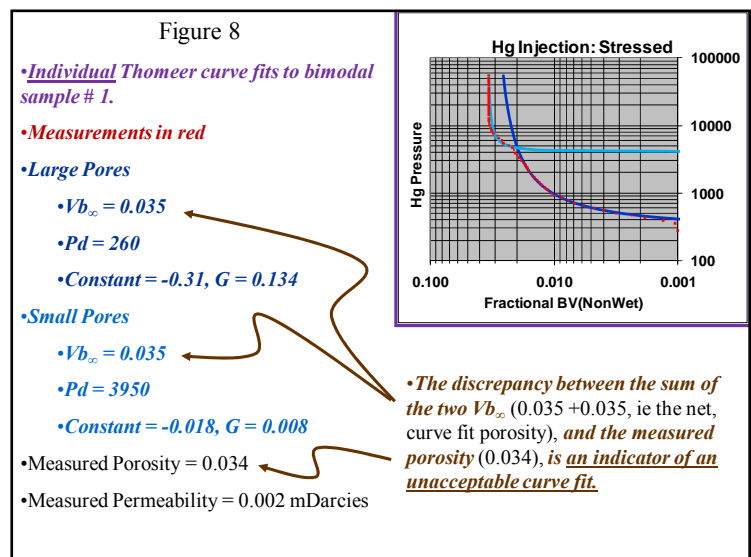
The expanded scale display, at lower right in the graphic, reveals the presence of a very small amount of a second pore system.

Figure 7 illustrates what is perhaps a more common issue, a relatively subtle transition from one pore system to another, across a wider range of capillary pressure / pore throat radii.

That is, there is a reasonably good match along the hyperbola asymptotes, but deviation in the apex area, as the actual rock measurements pass through a range of pore throat radii.

The interpreter must decide whether to describe the data with a single or double hyperbola, with a rule of thumb being (Ed Clerke, personal communication) that  $Vb_{\infty}$  should match the measured porosity to within  $\sim \pm 2$  pu.

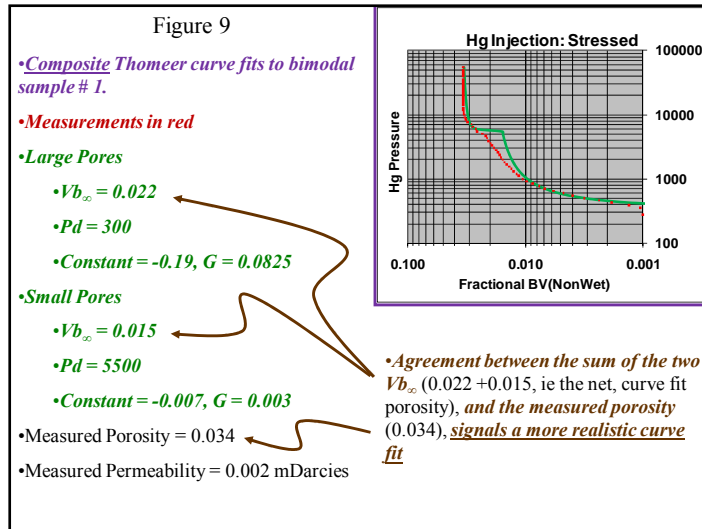
The third sample, Figure 8, is clearly a dual porosity response, and can be used to illustrate the curve fitting technique in those circumstances. Now, even though it is possible to closely represent the data with two independent hyperbola, that is not the physically meaningful solution as can be seen from the fact that the two  $Vb_{\infty}$  sum to 7 pu, while the measured porosity is only 3.4 pu.



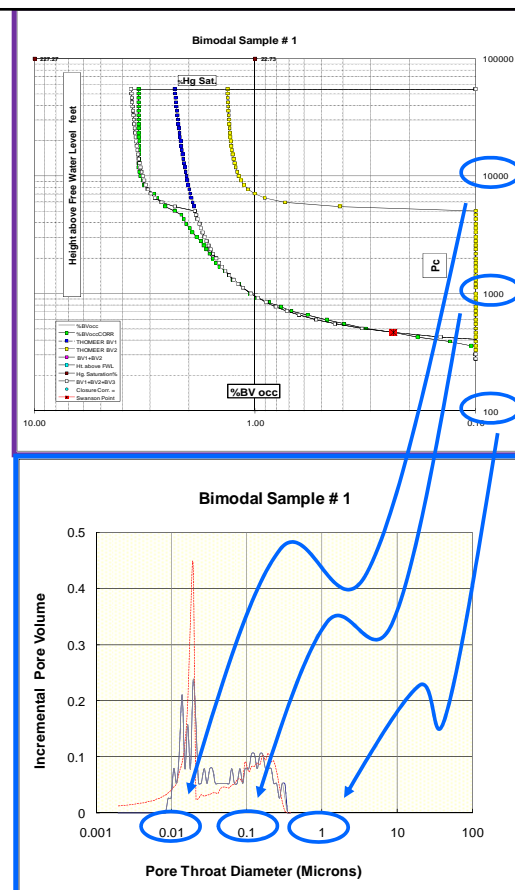
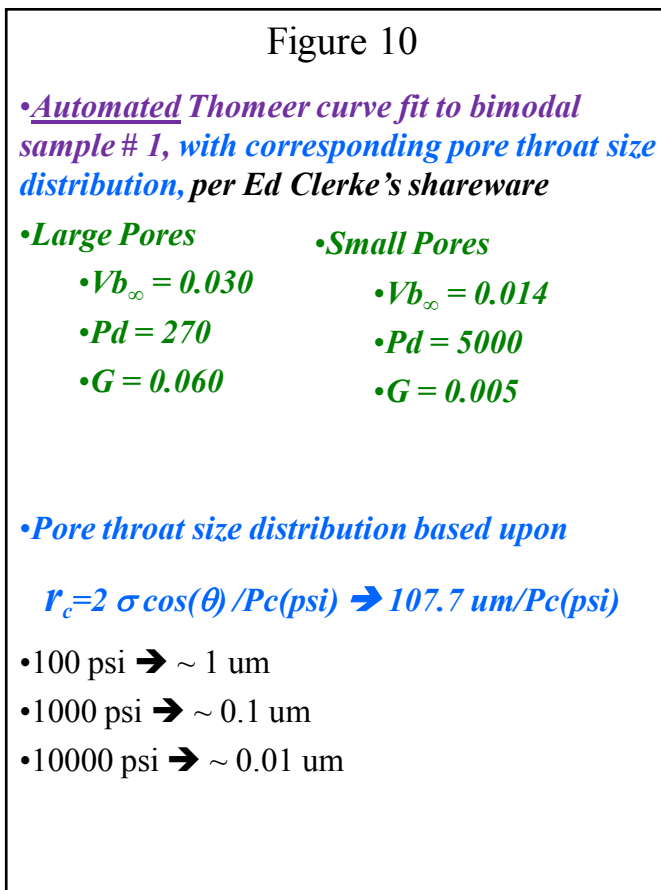
The representative, physically meaningful curve fit is the composite (superposition) of two hyperbola for which the net  $Vb_{\infty}$  is 3.7 pu, comparable to the measured 3.4 pu: Figure 9.

Honoring the porosity constraint also reveals the presence of a poorly sorted response between the two 'end point' hyperbolas, which might have gone unappreciated in the absence an analytical mathematical model.

The preceding curve fits were achieved by manual iteration, but Clerke and Martin (2004) distribute shareware which automates the process, and additionally displays the pore throat distribution: Figure 10.



While it's straight-forward and fairly easy to determine the parameters manually, the spreadsheet solution becomes attractive if the database is large, plus it offers the advantage of a consistent curve fit, one sample to the next.



## Beyond the Individual Capillary Pressure Curves

An initial objective of the capillary pressure analysis is characterization of the reservoir saturation – height response; where can hydrocarbons be expected and what will the saturation be?

An ultimate objective may very well be a quantitative reservoir rock type classification protocol, and the Thomeer formulation lends itself to that, as well (Ed Clerke, 2008). In the Arab D limestone, Thomeer's  $P_d$  is found to be a dominant descriptor, and present as four distinct modes.

By analyzing the  $P_c$  data within Thomeer's framework one is able to directly recognize locally specific modes and (pore throat) size ranges, in contrast to generic classifications which are suitable for analogue reference purposes, but are not necessarily the boundary values dominating a specific reservoir.

In the Arab D limestone the unique local characterization explains the large variation historically observed in the porosity – permeability crossplot; the micro-porous population does not contribute significantly to permeability. With 70% of the rock exhibiting multi-mode pore systems, failure to account for the non-contributing small pores leads to a large uncertainty.

When the porosity → permeability relation is cast in terms of the macro pore system displacement pressure ( $P_d$ ), an improved correlation is found. This approach is similar in concept to the underlying relation of the Lucia Petrophysical Classification protocol; displacement pressure and grain / crystal size are inter-related, and correspond to boundaries on the porosity – permeability crossplot.

The Thomeer formulation can also serve as an up-scaling vehicle (Ekrann, 1999 and Buiting, 2007), and in the Arab D leads to the recognition that grid blocks may begin to fill with hydrocarbon much closer to the free water level than would have been anticipated with a routine saturation-height approach.

### Summary

While mercury injection is a routinely utilized reservoir characterization tool, there is in fact often more information to be extracted, than may have been done.

By performing the interpretation within a standardized framework, one is able to more readily recognize the presence of an additional pore system, and to further deduce the mathematical relation which represents the  $P_c$  response.

The Thomeer descriptors ( $V_{b\infty}$ ,  $P_d$ ,  $G$ ) are physically meaningful and may be suitable for rock type clustering purposes.

Finally, at some point one is typically going to need to 'initialize' the static reservoir model, and the Thomeer formulation lends itself to up-scaling.

## Appendix 1: Properties of Thomeer Hyperbolae

Courtesy Ed Clerke

- A single pore system can be represented by one Thomeer hyperbolae and is completely characterized by just three numbers;  $P_d$ ,  $B_{v,\infty}$ ,  $G$ .
- The Thomeer hyperbolae relies upon no other attributes (with associated errors and uncertainties); it is self-contained.
- A Petrophysical Rock Type (PRT) can be defined as a cluster in Thomeer parameter space;  $P_d$ ,  $B_{v,\infty}$ ,  $G$ .
- Air permeability can be computed and predicted from the pore network parameters,  $P_d$ ,  $B_{v,\infty}$ ,  $G$ , to within a multiplicative uncertainty of 1.8x, and this can be compared to a measured permeability (as a Quality Control device).
- The Thomeer hyperbolae obey the law of superposition and can then be combined (superposed) to quantify complex pore systems.
- A Thomeer forward modeled capillary pressure curve can be generated from insight into the attributes which may come from a variety of sources of rock data; cores to cuttings to a Rock Catalog.

## Appendix 2: Thomeer Curve Fit Guidelines

Courtesy Ed Clerke

- Always try to fit the data using the least number of pore systems
- The signal for bimodality can be either
  - An obvious kink in the data, or
  - More subtly, a major mismatch of the BV(total) mono-modal against the measured porosity, when fit with an incorrect modality assumption (assumed mono-modal)
- In the case of a dual (or more) porosity system, it is the sum of the individual curve fits that should over-lay the measurements. Execute the individual curve fits sequentially, ensuring that the composite curve fit is matching the actual measurements.
- Curve fitting is best done with the Share Ware Excel spreadsheet, which utilizes the Solver function.
- The objective of the Thomeer spreadsheet is to optimize the curve fit within the context of the following criteria.
- Minimal Closure correction
- Best fit to MICP data
- Minimum number of pore subsystems
- BV total comparable to He Por +/- 2



- Computed Perm (Thomeer) to Actual Perm within 2x
- Sample image in good shape

### Appendix 3: Logarithms

In today's computerized world, the utility of logarithms may not be immediately obvious, but in their time they constituted a 'giant step forward' in a manner somewhat similar to the hand calculators and laptop computers in use today.

Logarithms can be defined with respect to any positive base, and will differ one base to the next by only a constant multiplier. Since our calculations are usually in a Base 10 number system (we have ten fingers and ten toes, and the human mind built upon that), that reference is one obvious choice.

$\text{Log}_{10}(x)$  is defined as is the power to which 10 must be raised, in order to yield the value  $x$ .

- $\text{Log}_{10}(1) = 0$ , since  $10^0=1$
- $\text{Log}_{10}(10) = 1$ , since  $10^1=10$
- $\text{Log}_{10}(100) = 2$ , since  $10^2=100$

Another *natural base* arises within the context of calculus, as the area under the curve  $f(x) = 1/x$ , from  $1 \rightarrow x$ . Now the base (reference) is the irrational number  $e \approx 2.718281828$ .

The utility of logarithms lies in the fact that multiplication of actual numbers is accomplished by addition of logarithms, and division of actual numbers corresponds to subtraction of their logarithms. One is then able to perform calculations much quicker, and with less chance of error.

Next, recognizing that multiplication is achieved with addition, we realize that by scaling two linear objects in an appropriate manner, multiplication may be done by adding the respective, appropriate lengths of the two numbers in question: the slide rule. The slide rule of yesterday is the analogue of the hand calculator of today.

In addition to simplifying multiplication and subtraction, logarithms are also attractive when dealing with equations that involve an exponential term, such as radioactive decay, etc and it is in this context (and others) that natural (Base  $e$ ) logarithms become attractive: hence the characterization of this base as 'natural'.

Base 10 and Base  $e$  logarithms differ only by a constant multiplier.

Number	$\text{Log}_{10}(x)$	$\text{Ln}(x)$	Ratio
1	0	0	$\text{Log}(x)/\text{Ln}(x)$
10	1	2.302585	0.43429448
100	2	4.60517	0.43429448
1000	3	6.907755	0.43429448

In the case at hand, the relation of interest is

$$V_b / V_{b\infty} = 10^{\{\text{Constant} / [\text{Log}(P_c / P_d)]\}} \rightarrow \exp[-G / \text{Log}(P_c / P_d)]$$

The conversion to Base e follows

$$[\text{Log}(V_b / V_{b\infty})] = \text{Constant} / [\text{Log}(P_c / P_d)]$$

$$\text{Log}(V_b / V_{b\infty}) = (1/0.4343)[\text{Ln}(V_b / V_{b\infty})]$$

$$\text{Ln}(V_b / V_{b\infty}) = 0.4343\{\text{Constant} / [\text{Log}(P_c / P_d)]\}$$

$$(V_b / V_{b\infty}) = \exp\{0.4343\{\text{Constant} / [\text{Log}(P_c / P_d)]\}\} = \exp[-G / \text{Log}(P_c / P_d)]$$

$$\Rightarrow -G = 0.4343 * \text{Constant}$$

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## Biography

R. E. (Gene) Ballay's **34 years in petrophysics** include **research and operations** assignments in Houston (Shell Research), Texas; Anchorage (ARCO), Alaska; Dallas (Arco Research), Texas; Jakarta (Huffco), Indonesia; Bakersfield (ARCO), California; and Dhahran, Saudi Arabia. His carbonate experience ranges from individual Niagaran reefs in Michigan to the Lisburne in Alaska to Ghawar, Saudi Arabia (the largest oilfield in the world).

He holds a **PhD in Theoretical Physics** with **double minors in Electrical Engineering & Mathematics**, has **taught physics in two universities**, **mentored Nationals** in Indonesia and Saudi Arabia, published **numerous technical articles** and been designated **co-inventor on both American and European patents**.

At retirement from the Saudi Arabian Oil Company he was the senior technical petrophysicist in the Reservoir Description Division and had represented petrophysics in three multi-discipline teams bringing on-line three (one clastic, two carbonate) multi-billion barrel increments. Subsequent to retirement from Saudi Aramco he established Robert E Ballay LLC, which **provides physics - petrophysics consulting services**.

He served in the US Army as a Microwave Repairman and in the US Navy as an Electronics Technician, and he is a USPA Parachutist and a PADI Dive Master.

