Rolling the Dice

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The only certainty in most of our formation evaluations is the presence of uncertainty and how that issue is (or is not) ***addressed***. At the simplest level one may estimate the Best and Worst Case, for each input attribute, and then bound the evaluation with the resulting extreme values, even as we recognize that the simultaneous occurrence of multiple "best" or "worst" values is an unlikely event.

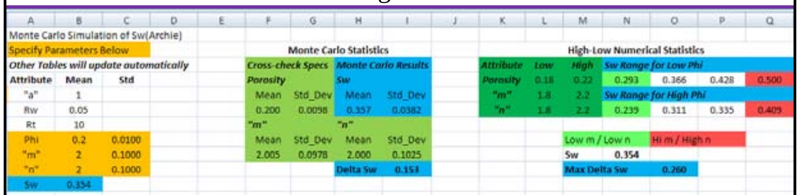
It is in fact ***relatively simple to address the uncertainty question in comprehensive, realistic and quantitative fashion***, and to further identify ***where to focus time, and money, in search of an improved evaluation.***

At the simplest level ***our Sw estimates are compromised by uncertainty in the various Archie equation attributes***.

$$S_w{}^n = a\, R_w / (\Phi^m R_t)$$

In an earlier article (Risky Business) we took the derivative of Archie's equation (the same approach will suffice for a shaly sand equation), and calculated the individual impact of each term's uncertainty upon S_w to identify where the biggest bang for the buck, in terms of a core analyses program or suite of potential logs, was to be found. At that time we noted the ***'link between parameters***, in that ***the relative importance of a single attribute, can be dependent upon the magnitude of another attribute, so that the characterization must be done for locally specific conditions.***

Figure 1



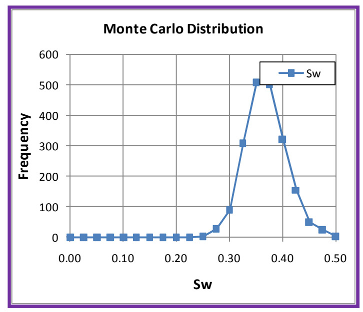
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
Monte Carlo Simulation of Sw(Archie)																
Specify Parameters Below					Monte Carlo Statistics					High-Low Numerical Statistics						
Other Tables will update automatically					*Cross-check Specs*		*Monte Carlo Results*			*Attribute*	*Low*	*High*	*Sw Range for Low Phi*			
Attribute	Mean	Std			*Porosity*		*Sw*			*Porosity*	0.18	0.22	0.293	0.366	0.428	0.500
"a"	1				Mean	Std_Dev	Mean	Std_Dev		"m"	1.8	2.2	*Sw Range for High Phi*			
Rw	0.05				0.200	0.0098	0.357	0.0382		"n"	1.8	2.2	0.239	0.311	0.335	0.409
Rt	10				"m"		"n"									
Phi	0.2	0.0100			Mean	Std_Dev	Mean	Std_Dev				Low m / Low n		Hi m / High n		
"m"	2	0.1000			2.005	0.0978	2.000	0.1025				Sw	0.354			
"n"	2	0.1000					Delta Sw	0.153				Max Delta Sw		0.260		
Sw	0.354															

•*Monte Carlo simulation of Sw(Archie) for above user-specified distributions*

•"a", R_w and R_t are assumed to be 'well known', but may be varied by a straight-forward extension of the technique

•With these specifications, ***there is a 95% probability that S_w is bounded by*** 0.357 +/– 2(0.0382) ➔ ***$0.28 < S_w < 0.43$***

•***The Best / Worst case would have $0.24 < S_w < 0.50$, the difference being the unlikely chance of all Best (or Worst) events occurring simultaneously***



2000 Simulations

An alternative approach is ***Monte Carlo*** simulation, which can be ***implemented with routine Excel spreadsheet functions***. The Monte Carlo method randomly assigns values, according to ***user specified probability distributions***, to each of the input parameters and then calculates the result. When the simulation is repeated a statistically significant number of times (results herein are based upon 2000 passes, which Excel handles without a problem), one is ***able to determine the likely outcome within any specific probability band, and to further identify which parameter is dominating the uncertainty*** (and hence where time and money is most efficiently directed for an improved result).

As an example, with the specifications tabulated in Figure 1, there is a ***95% probability*** (+/- two standard deviations) that ***$0.28 < S_w < 0.43$, whereas the Best / Worst approach would bound the results with $0.24 < S_w < 0.50$; the difference being the unlikely event of multiple,***

simultaneous Best or Worst events. Not only does ***Monte Carlo give us a more realistic summary***, but ***by varying the input standard deviations (uncertainties), one is able to identify where to most efficiently concentrate time / money in an effort to improve results.***

Monte Carlo Technique

The ***Monte Carlo*** method relies on repeated random sampling of user specified input probability distributions to model expected results. This approach is ***attractive when it is infeasible or impossible to compute an exact result with a deterministic algorithm***.

An ***advantage of Monte Carlo*** is that any type of distribution can be used to characterize the uncertainty specification of input parameters, for example normal, log normal, etc; an issue since the phenomena governing frequency distributions in nature often favor log-normal (Limpert et al, 2001).

As illustrated in Figure 1, Monte Carlo also allows one to quantify the upside and downside better than a Best / Worst approach, and to recognize which distribution (parameter) is dominating the result uncertainty.

A ***limitation of Monte Carlo*** is that special software is typically utilized (commercial add-ons to Excel, etc), and is often not even an option in commercially available petrophysics s/w packages. Common oilfield distributions, however, such as Normal, Log Normal and Triangle are available in Excel and it is straight-forward to implement Monte Carlo within the Excel framework. In this approach, one remains in the familiar Excel environment, and actually leverages their Excel skill set via the additional hands-on experience within the platform.

A discussion of the Monte Carlo method can be found in Decision Analysis for Petroleum Exploration by Paul Newendorp & John Schuyler, and a collection of articles addressing exploration risk can be found in The Business of Petroleum Exploration published by the AAPG, Tulsa, Oklahoma.

Additional information may be found in the References, with useful on-line reference material to be found at the following links.

- http://www.enrg.lsu.edu/pttc/
- http://www.mrexcel.com/
- http://people.stfx.ca/bliengme/exceltips.htm
- http://office.microsoft.com/en-us/excel/HA011118931033.aspx
- http://www.statsoft.com/textbook/stathome.html
- http://en.wikipedia.org/wiki/Monte_carlo_simulation
- http://www.ipp.mpg.de/de/for/bereiche/stellarator/Comp_sci/CompScience/csep/csep1.phy.ornl.gov/mc/mc.html
- http://www.sitmo.com/eqcat/15
- http://www.riskglossary.com/link/monte_carlo_method.htm
- http://www.chem.unl.edu/zeng/joy/mclab/mcintro.html

The Gaussian or Normal Distribution

For illustration purposes, we focus here on the bell shaped Gaussian distribution. Log Normal or Triangular distributions are easily handled with a simple change of Excel functions.

Gauss was a child prodigy and perhaps the greatest mathematician since antiquity (http://en.wikipedia.org/wiki/Carl_Friedrich_Gauss). When the dwarf planet Ceres was discovered, and observed for only a few days before vanishing, Gauss was able to mathematically predict where it would be found a year later, and missed the mark by only half a degree.

The bell shaped, or normal, probability distribution, is the most widely used family of statistical distributions and came to be referred to as Gaussian because he analyzed astronomical data within that context.

Two parameters characterize the Gaussian distribution, the 'mean' and 'variance': Figure 2

Figure 2

•***Petroleum applications*** typically use "***normal***", "***log normal***", and "***triangular***" statistical ***distributions.***

•Probably the ***best known statistical distribution*** is the "***bell shaped***" ***normal distribution***, whose ***probability density function*** is described by:

$$\boxed{f_N(x) \quad = \quad \frac{1}{\sigma\sqrt{2\pi}} \times e^{\left\{-(x-\mu)^2 / 2\sigma^2\right\}}}$$

•***Two parameters characterize the distribution***

•the "***mean***" value μ,

•the ***variance*** represented by σ^2.

•The square root of the variance is the ***standard deviation***.

http://www.enrg.lsu.edu/pttc/

Excel's NormDist function [NORMDIST(x, mean, standard_dev, ***cumulative***)] calculates both the probability density, and cumulative probability, for a specified "x" value with given mean and standard deviation (standard deviation being the square root of the variance): Figure 3.

Setting the logical variable "***cumulative***" to "***false***" in the preceding expression will yield the "probability mass function", while setting "***cumulative***" to "***true***" returns the "cumulative" distribution.

Be aware that Excel 2007 is used for these illustrations and while there is a Compatibility Mode for earlier Excel versions, it is conceivable that screens could differ. For greater clarity, the graphics / text in Figure 3 are color coded, and in each case (true & false) the Excel cursor has been placed in a calculation cell, so that the functional form of NormDist is displayed at the top of the respective screen capture.

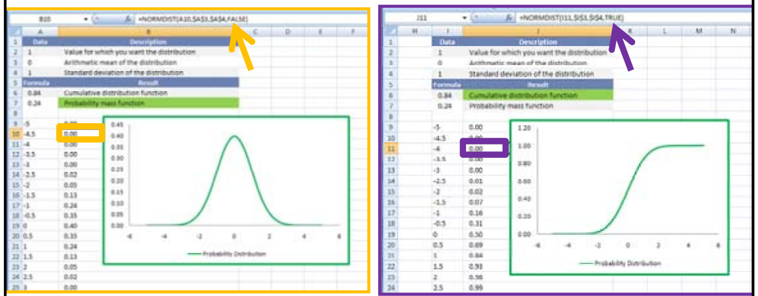
As an illustrative interpretation of the cumulative distribution, we recognize (Figure 3, right side) that the Cumulative Probability (vertical axis) has reached 0.50 (50%) when the Probability Distribution (horizontal axis) is 0. As expected, when the mean value is specified to be 0, there is equal probability of any single value being higher, or lower.

Departing from the mean, Figure 3 (graphical display and Excel cell values) reveal that a cumulative probability of

- 16% has been reached @ x= -1.0,
- 31% @ x=-0.5,
- 69% @ x=+0.5,
- 84% @ x=1.0, etc.

There is then a 31% chance that "x" is at least -0.5 in magnitude, a 69% chance that "x" is at least 0.5 in magnitude, etc.

Figure 3



•Syntax: ***NORMDIST***(x, mean, standard_dev, ***cumulative***)

•There are ***two variations of NORMDIST, according to the value assigned cumulative***

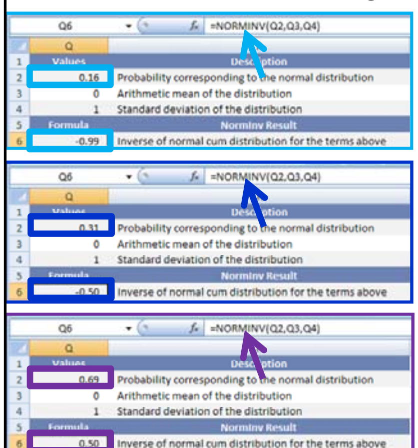
•***Cumulative is a logical parameter*** that determines the form of the function

•If ***True, it returns the cumulative distribution function.***

•If ***FALSE, it returns the probability mass function.***

The NORMINV [NORMINV(Probability, Mean, Standard_Dev)] performs the inverse operation by returning the "x" value for a given cumulative probability of normal distribution with specified mean and standard deviation: Figure 4.

Figure 4



•The NORMINV [NORMINV(Probability, Mean, Standard_Dev)] performs the inverse operation by returning the "x" value for a given cumulative probability of normal distribution with specified mean and standard deviation

•Cumulative probability is 16% when "x" attains the value -0.99

•Cumulative probability is 31% when "x" attains the value -0.50

•Cumulative probability is 69% when "x" attains the value 0.50

This calculation indicates the Cumulative probability is

- 16% when "x" attains the value -0.99,
- 31% when "x" attains the value -0.50
- 69% when "x" reaches 0.50.

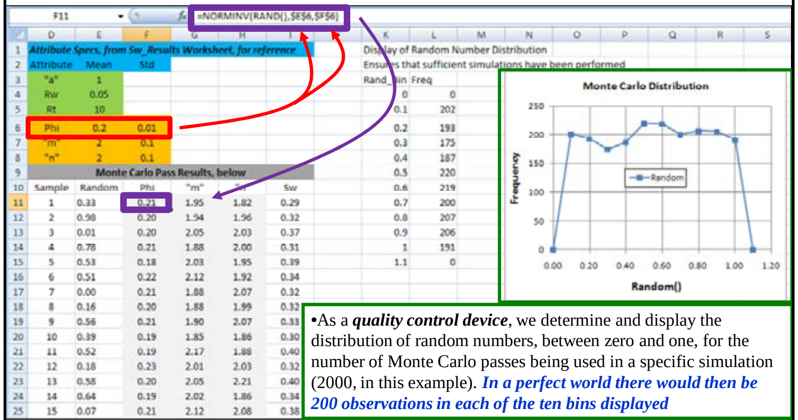
Monte Carlo Modeling of Sw(Archie)

With a basic understanding of what the Excel Gaussian Distributions options are, one is able to model the Archie equation within that framework. For illustration purposes, we regard "a", R_w and R_t to be well-known, and Φ, "m" and "n" subject to uncertainty as specified in Figure 5. Allowance for uncertainty in "a", R_w and R_t may be addressed by a straight-forward extension of the techniques presented here. Also, ***while the focus here is on the simple Sw(Archie), any other model*** (shaly sand, for example) ***may be evaluated in a similar manner.*** Once the concepts are understood, locally specific models are readily developed.

Each of the uncertain attributes is modeled as a random number input to NormInv, whose mean value and standard deviation are locally appropriate. For example (Exhibit 5), the first pass random estimate of porosity, with a distribution centered on 20 pu and having a standard deviation of 1 pu, results in an estimate of 21 pu. The random values of “m” and “n”, appropriate to the specified distributions, are independently and randomly determined, and S_w calculated per the Archie relation.

Because Excel recalculates equations each time the spreadsheet is opened, or specifications are changed, the various results will change (your line item spreadsheet values will change, each time you make a modification).

Figure 5

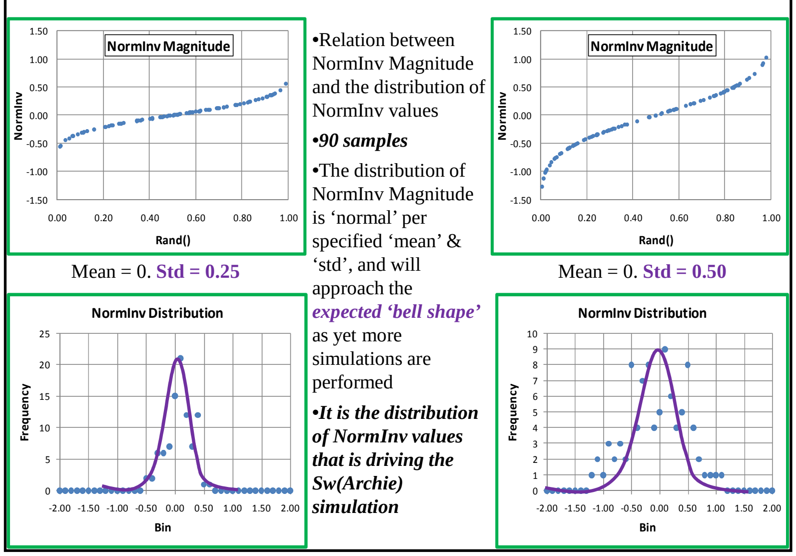


•Each of the uncertain attributes are modeled as a random number input to NormInv, whose mean value and standard deviation are locally appropriate. For example, ***the first pass random estimate of porosity, with a distribution centered on 20 pu and having a standard deviation of 1 pu, results in an estimate of 21 pu.***

As a ***quality control device***, we determine and display the distribution of random numbers, between zero and one, for the number of Monte Carlo passes being used in a specific simulation (2000, in this example). ***In a perfect world there would be 200 observations in each of the ten blue bins displayed in Figure 5***.

Figure 6 illustrates the relation between the magnitude of NormInv, and the distribution of NormInv values, for different standard deviations, at 90 simulations. Both distributions take on an approximate Gaussian appearance, with the larger standard deviation result displaying more scatter. ***It is the distribution of NormInv values that is driving the Sw(Archie) simulation***. It’s important to realize that each occurrence of NormInv involves an independent Rand() input.

Figure 6



Mean = 0. Std = 0.25

Mean = 0. Std = 0.50

•Relation between NormInv Magnitude and the distribution of NormInv values

•90 samples

•The distribution of NormInv Magnitude is ‘normal’ per specified ‘mean’ & ‘std’, and will approach the ***expected ‘bell shape’*** as yet more simulations are performed

•It is the distribution of NormInv values that is driving the Sw(Archie) simulation

The ***approach taken here is intended to parallel that of the LSU results*** (***Must Read supplemental material***), which also includes Log Normal and Triangle distributions, and so can be directly referenced if either of those distributions are required: ***www.enrg.lsu.edu/pttc/.***

As an additional QC device, the statistical attributes of the simulated quantities (Φ, 'm' and 'n' in this example) are tabulated ***directly from the simulation population***, and displayed graphically: Figure 7.

With 2000 simulations, the model population nicely replicates the input numerical specifications, and the porosity distribution takes on the expected appearance (Figure 7).

Simulation results are reported both numerically and graphically: Figure 8. In this particular case, there is ***a 95% likelihood that Sw is contained within + / - 2 σ***, (0.357 – 0.076) < Sw < (0.357 + 0.076) ➔ 0.***28 < Sw < 0.433***.

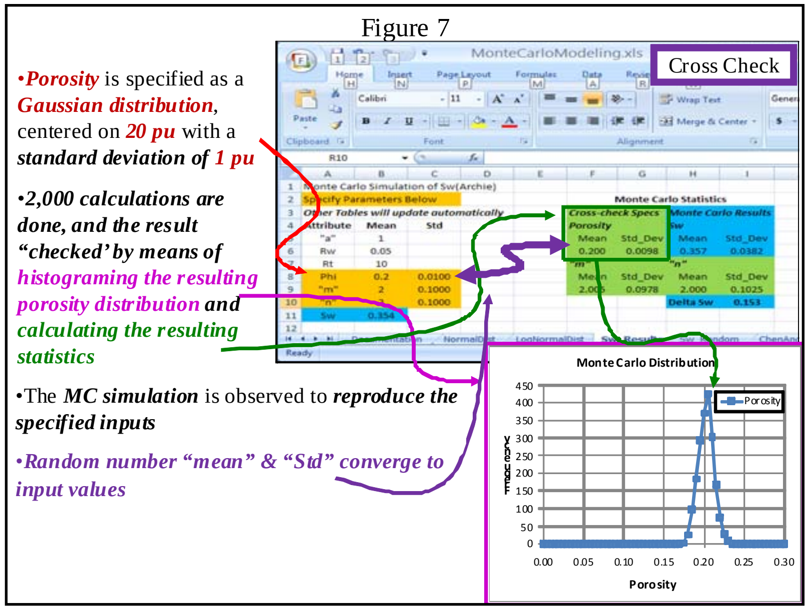


Figure 7

In utilizing Excel frequency distribution graphics, one should ***take note of how the 'bins' are populated, as they are not 'centered'***. This can cause the graphic to take on a shifted appearance, with respect to the numerical report (consult Excel Help on the Frequency function for details).

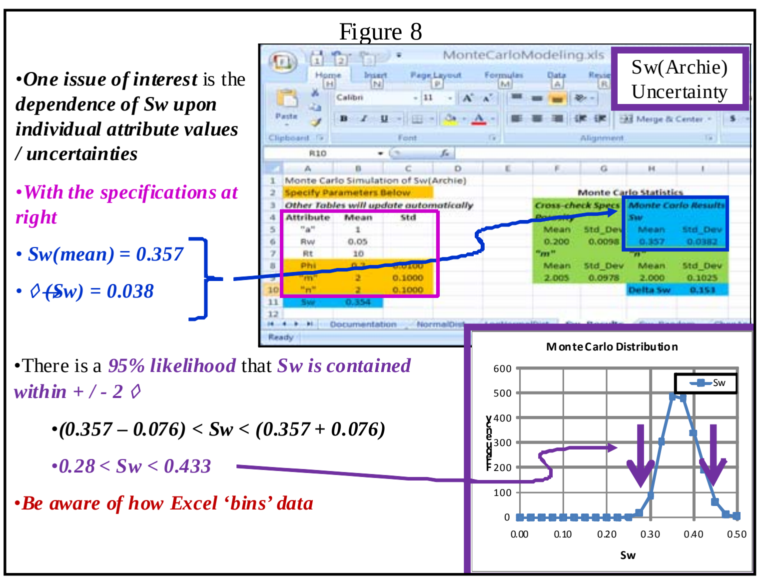


Figure 8

The Sw(Archie) ***result population is further affected by the nonlinear relation between the various attributes, as discussed by Bryant et al in Understanding Uncertainty, Oilfield Review. Autumn 2002***, who illustrates that ***a normal uncertainty distribution about a given porosity yields a log-normal distribution for the resulting Sw distribution.*** ***Bryant's article is another Must Read***.

Figure 9

- + / - 1 σ will encompass ~ 68% of the distribution
- +/- 2 σ ~ 95 % of the distribution,

•*Approximate the "Phi" relative uncertainty* of 15% @ 10 pu (1.5 pu) as 2 σ ~ 1.5 pu => ***σ ~ 0.75 pu for Monte Carlo Simulation purposes***

•That is, ***2 σ encompasses 95 % of the spread in the distribution***, and it is ***set equal to the 15 % uncertainty of the Chen & Fang analyses***

•***Approximate the "m" relative uncertainty*** of 10% @ 2.00 (0.2) as 2 σ ~ 0.2 => ***σ ~ 0.1 for Monte Carlo Simulation purposes***

•***Approximate the "n" relative uncertainty*** of 5% @ 2.00 (0.1) as 2 σ ~ 0.1 => ***σ ~ 0.05 for Monte Carlo Simulation purposes***

•***The Biggest Bang For The Buck is to be found in "m" as it has the greatest uncertainty relative to the other attributes***

Attribute Uncertainties Specified Individually
Light Green Cells require User Specification
Light Blue Cells are calculated results

Attribute	Individual Uncertainty	Best Estimate	Relative Uncertainty On Sw(Archie)
a	0.0%	1.00	0.0000
Rw	0.0%	0.02	0.0000
Phi	15.0%	0.10	0.0900
m	10.0%	2.00	0.2121
n	5.0%	2.00	0.0224
Rt	0.0%	40.00	0.0000
Sw		22%	

The Biggest Bang for the Buck
Derivatives vs Monte Carlo

After C. Chen & J. H. Fang. Sensitivity Analysis of the Parameters in Archie's Water Saturation Equation. The Log Analyst. Sept – Oct 1986

We are ***typically confronted with two issues***, first to ***characterize the uncertainty in the Sw*** estimate itself, and next to ***identify where time and money would be best spent to reduce that uncertainty***. There are ***two ways to proceed***: 1) ***take the derivative*** of Archie's equation with respect to each term, and compare the magnitude of each term against one another, for specific attribute values. 2) ***Monte Carlo simulation***, with the input attribute distributions specified per locally representative requirements.

The differential approach was illustrated in an earlier article, Risky Business, and those results have been included in the following so as to both 'make the connection' and to also serve as a cross-check. The illustrative attribute values / uncertainties are those in the Chen & Fang (1986) paper, so as to allow reference to that material as well.

In the case of Figure 9 attributes, ***the differential approach would indicate that time / money would be best spent on "m"***, as the relative uncertainty of the cementation exponent is far greater than any of the other attributes.

The connection between derivatives and Monte Carlo is made by recognizing that two standard deviations encompasses 95% of the statistical scatter, and then setting, attribute by attribute, 2σ equal to the Chen & Fang illustrative uncertainties, thereby forming the ***Base Case*** for Monte Carlo: Figure 9.

Monte Carlo simulations are then performed, incrementally, with each attribute better defined by 10% and the improvement (reduced scatter) in the resulting Sw noted: Figure 10.

Figure 10

•**Chen & Fang** identify the ***attribute resulting in the greatest Sw uncertainty***

•In the case at ***right***, ***"m" is the dominant issue***

•This ***same issue can be addressed with Monte Carlo simulation*** (below) ***Derivatives vs Monte Carlo***

Attribute Uncertainties Specified Individually
Light Green Cells require User Specification
Light Blue Cells are calculated results

Attribute	Individual Uncertainty	Best Estimate	Relative Uncertainty On Sw(Archie)
a	0.0%	1.00	0.0000
Rw	0.0%	0.02	0.0000
Phi	15.0%	0.10	0.0900
m	10.0%	2.00	0.2121
n	5.0%	2.00	0.0224
Rt	0.0%	40.00	0.0000
Sw		22%	

•The ***Base Case*** is at ***lower left***, with ***each simulation towards the right reflecting an improvement in "Phi", "m" and "n" individual precision by 10 %, respectively.***

Attribute	Mean	Std
"a"	1	
Rw	0.02	
Rt	40	
Phi	0.1	0.0075
"m"	2	0.1000
"n"	2	0.0500

Monte Carlo Results Sw

Mean	Std_Dev
0.227	0.0332

Attribute	Mean	Std
"a"	1	
Rw	0.02	
Rt	40	
Phi	0.1	0.0068
"m"	2	0.1000
"n"	2	0.0500

Monte Carlo Results Sw

Mean	Std_Dev
0.225	0.0323

Attribute	Mean	Std
"a"	1	
Rw	0.02	
Rt	40	
Phi	0.1	0.0075
"m"	2	0.0900
"n"	2	0.0500

Monte Carlo Results Sw

Mean	Std_Dev
0.226	0.0297

Attribute	Mean	Std
"a"	1	
Rw	0.02	
Rt	40	
Phi	0.1	0.0075
"m"	2	0.1000
"n"	2	0.0450

Monte Carlo Results Sw

Mean	Std_Dev
0.226	0.0329

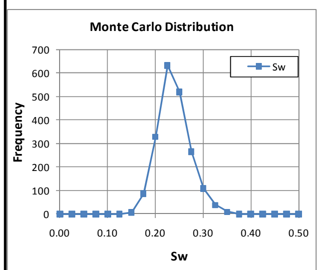
Monte Carlo simulation reveals that ***a 10% improvement in definition of the cementation exponent will yield the greatest reduction in Sw uncertainty***, relative to the other attributes, and consistent (as expected) with Chen and Fang (Figure 9).

Figure 11

Attribute	Mean	Std
"a"	1	
Rw	0.02	
Rt	40	
Phi	0.1	0.0075
"m"	2	0.1000
"n"	2	0.0500
Sw	0.224	

•One ***also observes that the Best / Worst numerical evaluation of Sw(Archie) is considerably more pessimistic than is the +/- 2 σ Monte Carlo simulation***

•The Best and Worst of all parameters are unlikely to occur simultaneously



Monte Carlo Statistics

Cross-check Specs		Monte Carlo Results	
Porosity		Sw	
Mean	Std_Dev	Mean	Std_Dev
0.100	0.0076	0.226	0.0333
"m"		"n"	
Mean	Std_Dev	Mean	Std_Dev
1.997	0.1005	2.001	0.0508
		Delta Sw	0.133

High-Low Numerical Statistics

Attribute	Low	High	Sw Range for Low Phi			
Porosity	0.085	0.115	0.189	0.222	0.318	0.355
"m"	1.8	2.2	Sw Range for High Phi			
"n"	1.9	2.1	0.142	0.171	0.224	0.258

Low m / Low n	Hi m / High n
Sw	0.224
Max Delta Sw	0.212

Best Case
Worst Case

We also note that the Best / Worst case scenario would significantly over-state the 95% Monte Carlo uncertainty, because it's unlikely (though not impossible) that the Best or Worst, of all attributes, would occur simultaneously: Figure 11.

Were porosity to be 25 pu, rather than the 10 pu of the above example, the focus changes. Now attention is best devoted to the "n" exponent: Figure 12.

The uncertainty in an Sw estimate is a dynamic issue, dependent upon the relative magnitudes of the input attributes which are themselves linked, and thus may change through the reservoir.

Figure 12

•If porosity were to be 25 pu, rather than 10, the focus changes

•"m" and "n" uncertainties have been set equal in this simulation

•The ***Base Case*** is at ***lower left***, with ***each simulation towards the right reflecting an improvement in "Phi", "m" and "n" precision by 10 %.***

•With the improved porosity, focus shifts to "n", the tortuosity of the conductive (brine) phase in the presence of a non-conductive (hydrocarbon) phase, as offering the Biggest Bang For The Buck.

•Improved "n" definition yields the greatest reduction in the standard deviation of Sw

Attribute	Mean	Std
"a"	1	
Rw	0.02	
Rt	40	
Phi	0.25	0.0100
"m"	2	0.1000
"n"	2	0.1000
Sw	0.089	

Monte Carlo Results	
Sw	
Mean	Std_Dev
0.090	0.0132

Attribute	Mean	Std
"a"	1	
Rw	0.02	
Rt	40	
Phi	0.25	0.0090
"m"	2	0.1000
"n"	2	0.1000
Sw	0.089	

Monte Carlo Results	
Sw	
Mean	Std_Dev
0.090	0.0128

Attribute	Mean	Std
"a"	1	
Rw	0.02	
Rt	40	
Phi	0.25	0.0100
"m"	2	0.0900
"n"	2	0.1000
Sw	0.089	

Monte Carlo Results	
Sw	
Mean	Std_Dev
0.090	0.0127

Attribute	Mean	Std
"a"	1	
Rw	0.02	
Rt	40	
Phi	0.25	0.0100
"m"	2	0.1000
"n"	2	0.0900
Sw	0.089	

Monte Carlo Results	
Sw	
Mean	Std_Dev
0.090	0.0121

Uncertainty Specification

Uncertainty arises from multiple sources, and includes (among others) the following.

- Calibration data (routine grain density & porosity, special core analyses for "m" & "n", etc)
- The down hole instruments which are making the actual measurements
- The assumed interpretive model

Figure 13. Core Analyses Uncertainty

•The ***samples are assumed to not change between tests***, so that ***differences reflect random variations in the measurement process***

Measurement	Experience-based	Statistically derived[1]
Porosity	±0.5 Por %	±0.23%
Grain density	±0.01 g/cc	±0.0093 g/cc
Air permeability		
0.01–0.1md	±30%	±21%
0.1–1.0 md	±25%	±21%
1.0–50 md	±15%	±13%
50 md–1 darcy	±15%	±8%

•Deviations are interpreted as Gaussian in nature, so that

•+/- 1 σ encompasses 68% of the data,

•+/- 2 σ encompasses 95 % of the data

•Tabulated confidence limits reflect the 99% limits

•A single measurement made on the same sample, which falls outside the specified 99% level is likely to be in error

The Log Analyst 30, No 2, March – April 1989
A Statistical analysis of the Accuracy and Reproducibility of Standard Core Analysis. David C Thomas and Virgil J Pugh

Thomas (1989), of Amoco, performed a ***detailed study of the accuracy and reproducibility of repeated routine core analyses***, and provided the resulting 99% confidence limit: Figure 13. In practice, the issue is broader than just the individual core measurements, and also includes compaction adjustments to the core, volumetric differences between the core and log measurements, etc.

In addition to the core calibration data, there is also uncertainty in the borehole wireline / LWD measurements: Figure 14.

In practice, particularly in a field when there is legacy data present, these specifications will change with time and tool type. The 6FF40 induction tool, for example, had a skin effect issue below about 1 ohm-m and a signal-to-noise limit above ~ 100 ohm-m. Newer tools will have different limitations.

Figure 14. Wireline Tool Uncertainty

SERVICE NAME	TOOL	MEASUREMENT ERRORS
Dipole Shear Sonic Imager (DSI)	DSST-A	P&S Compressional DT: 2% P&S Shear DT: 3% Dipole Shear DT: 5% Stoneley DT: 3%
Compensated Neutron (CNL)	HGNS or CNL	0 – 20 pu +-1 pu 30 pu +-2 pu >45 pu +-6 pu
Litho-Density	LTD-D	RHOB: 1.6 – 3.0 g/cm3 +- 0.01 g/cm3 1– 1.6 g/cm3 +-0.02 g/cm3 PEF: 1.4 – 6.0 barne/e +-6% Maximum Logging Speed 1800 fph
Integrated Porosity Lithology Tool	IPLT	
Accelerator Porosity Sonde	APS	< 7 pu +/- .5 pu 7 – 30 pu +/- 7% 30 – 60 pu +/- 10%
Litho-Density Sonde	LDS	RHOB: 2.0 – 3.0 g/cc +/- .01 g/cc PEF: 1- 6 +/- 10%
Platform Express		
Compensated Neutron (CNL)	PEX-CNL	0 – 20 pu +-1 pu 30 pu +-2 pu >45 pu +-6 pu
Three-detector Litho-Density from Platform Express	PEX-TLD	RHOB: 0.01g/cm3 (no mudcake) PEF : 0.15 barne/e (no mudcake)

Courtesy Tim Pritchard, BG Group
Illustrative Values. Consult your Service Company for locally specific attributes.

The ***Archie exponents*** present an additional set of issues. Focke and Munn (1987) nicely illustrate the dependence of 'm' upon carbonate pore geometry, while 'n' is controlled by wettability (Sweeny & Jennings, 1960) and surface roughness (Diederix, 1982). In carbonates, wettability (and hence "n") may vary with pore size (Chardac et al, 1997), and pose an additional challenge, particularly in the transition zone.

Adams (2005) cautions (and illustrates) that quantitative ***uncertainty definition is more than just using Monte Carlo simulation to vary the inputs to the interpretation model. The largest source of uncertainty may be the interpretation model itself***.

Griffiths (2006) brings our attention to the ***challenges posed by carbonate dual porosity systems*** and potential electrical 'short circuits'.

Carlos Torres-Verdin observes "my experience shows that the ***biasing of apparent resistivity curves due to post-processing*** (eg deconvolution) can be more detrimental to uncertainty than Archie's parameters, with a conspicuous example being thin, hydrocarbon saturated intervals experiencing vertical resolution and invasion effects.

Voss (1998) comments on determination of uncertainty ranges as does Bowers (2003). ***Summary points*** include

- A single interpreter should avoid making estimates on their own.
    - A single interpreter often lacks the needed knowledge to correctly estimate every parameter.
    - In addition, many interpreters have a bias that smaller errors are better and they will appear more knowledgeable about the subject.
- The error must reflect the level of knowledge about the parameters and the data quality.
- ***A standard set of uncertainty ranges must be avoided*** because there is no standard situation in which to apply them.
- ***Unusual events also pose special problems***.
    - Most people have a better recall of unusual events
        - Therefore a tendency to overestimate the probability of such an event
        - Especially if that event occurred recently
- Another very common mistake is to allow a very small amount of data to quantify the range of uncertainty
    - If data sets are small, the ranges probably need to be increased.
- Boundary Conditions
    - Water saturation must lie between zero and one
    - If the saturation values are too large or too small, the "best guesses" and ranges must be reconsidered and calculations remade.
- ***The final and probably most difficult problem to overcome is the culture and preconceived ideas of an organization.***

Methods and ranges of uncertainty applied to any analysis must be questioned every time they are applied.

These considerations (and others, included in the References) are not meant to be overwhelming, but rather simply realistic. Each case may very well be different, and must be addressed on an individual basis.

Summary

There are ***two basic ways in which the issue of uncertainty can be characterized***; ***partial derivatives*** of the expression of interest (Sw in this situation) and ***Monte Carlo simulation***. At the simplest level, ***they complement one another***, and since each are easily coded into an Excel spreadsheet, we routinely perform both, as a QC cross-check.

The ***deterministic derivative approach*** yields an equation, which ***may be easily coded into foot-by-foot petrophysical analyses***, in those cases for which the commercial petrophysics s/w does not include an uncertainty characterization option. One is then able to 'bound' the calculated results, foot-by-foot, which is an improvement over a 'generic envelope', given the inter-dependence of the result and specific reservoir values. An illustration of this method may be found in Ballay, Risky Business, March 2005, www.GeoNeurale.com

On the other hand, an attribute specific distribution may not be Gaussian. Focke and Munn, for example, investigated the dependence of the cementation exponent upon pore geometry. Suppose across a given interval we are unable to distinguish between interparticle and vuggy porosity; either is a possibility. The associated "m" distribution could then be rectangular, not Gaussian, ***an issue that the Monte Carlo approach can easily address*** (each input attribute can have its specific distribution, independent of the others).

In any case, it's ***important to recognize the following.***

- The uncertainty in Sw(Archie), and other common oilfield calculations, can be quantitatively addressed by both differential and statistical modeling approaches.
- Excel can handle common probability distributions, and can then serve as the Monte Carlo simulator. The derivative method will yield equations which may be easily coded into Excel, thereby facilitating a cross-check.
- Quantitative estimation of the uncertainty allows one to determine where time / money is most effectively spent, and to further avoid the trap of being misled as a result of a previous bad experience with a poorly defined parameter
- The importance of the various input parameters will change, according to the various magnitudes. There may be a linkage in that one parameter becomes more or less important as another parameter value is change. One size does not fit all feet.
- The equations resulting from the derivative approach may be coded, foot-by-foot, into the petrophysics s/w package, thereby providing a live-linked uncertainty estimate to the actual, local reservoir properties.

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Omissions, typos etc remain, of course, my responsibility.

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Biography

R. E. (Gene) Ballay's 32 years in petrophysics include research and operations assignments in Houston (Shell Research), Texas; Anchorage (ARCO), Alaska; Dallas (Arco Research), Texas; Jakarta (Huffco), Indonesia; Bakersfield (ARCO), California; and Dhahran, Saudi Arabia. His carbonate experience ranges from individual Niagaran reefs in Michigan to the Lisburne in Alaska to Ghawar, Saudi Arabia (the largest oilfield in the world).

He holds a PhD in Theoretical Physics with double minors in Electrical Engineering & Mathematics, has taught physics in two universities, mentored Nationals in Indonesia and Saudi Arabia, published numerous technical articles and been designated co-inventor on both American and European patents.

At retirement from the Saudi Arabian Oil Company he was the senior technical petrophysicist in the Reservoir Description Division and had represented petrophysics in three multi-discipline teams bringing on-line three (one clastic, two carbonate) multi-billion barrel increments. Subsequent to retirement from Saudi Aramco he established Robert E Ballay LLC, which provides physics - petrophysics consulting services.

He served in the U.S. Army as a Microwave Repairman and in the U.S. Navy as an Electronics Technician, and he is a USPA Parachutist and a PADI Dive Master.

