

The Law of Averages

R E (Gene) Ballay, PhD

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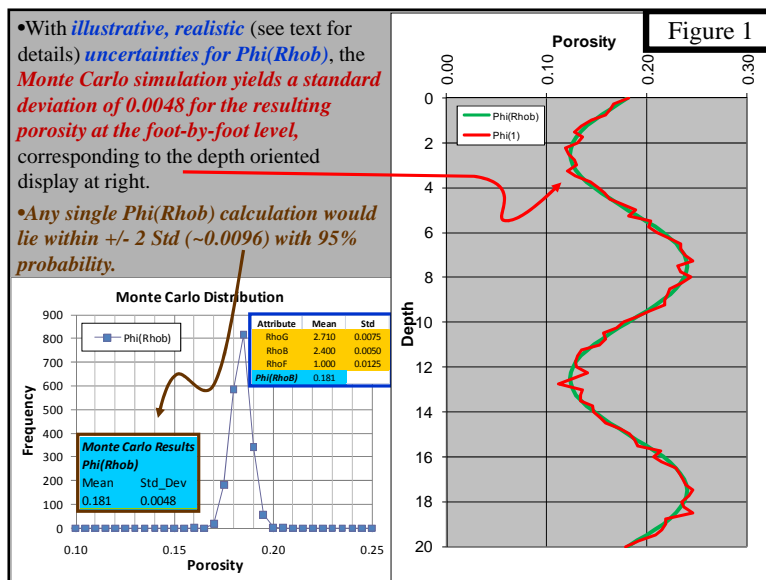
It has happened to most of us; the light is yellow as we approach the intersection, but since we are in a hurry we push on through, only to hear our passenger (significant other, parent, etc) comment “the law of averages is gonna catch you “.

There is, however, a bright side to “the law of averages” in that *so long as the “noise” is random, the uncertainty associated with a single foot-by-foot calculation is reduced at the layer average level*, and our *average results may be considerably better quantified than is a single foot-by-foot estimate*.

Our attention is typically focused on foot-by-foot calculations (and associated “noise”) *and there is a tendency to over-look the difference and regard the average values* (which will be used for simulator initialization, reserves estimation, etc) *as being subject to the same uncertainty as the foot-by-foot values, when in fact the layer averages may be significantly better known*.

In most evaluations, the Log Repeat seldom receives any attention beyond possibly a simple comment such as ‘repeat looks reasonable’. Were we to *take the time to digitally load the Repeat and compare it to the Main Pass in both the foot-by-foot and average value sense*, we would not only be able to *better QC each logging run individually*, but we could *also estimate the uncertainty present in the layer average values*.

The situation can be *illustrated with a physically realistic Monte Carlo simulation of Phi(Rhob)*: Figure 1. At the foot-by-foot level the statistical ‘noise’ in the model corresponds to a standard deviation of 0.0048, which infers that *any single calculation will be within +/- 2 * (0.0048) of the actual value, with 95 % probability*.



In practice it is desirable to model the logging tool response with locally appropriate attributes, and compare those results to the measured values (Main vs Repeat). *If time does not allow construction of the model, then valuable information is still available by simply establishing the empirically observed foot-by-foot difference distribution, for each well*. An unusually wide distribution is then a Red Flag, requiring an investigation.

The implications of the uncertainty at the layer average level can be illustrated and understood by imposing the modeled (or empirically determined) standard deviation of the noise upon a sine curve (or an actual Main Pass logging trace).

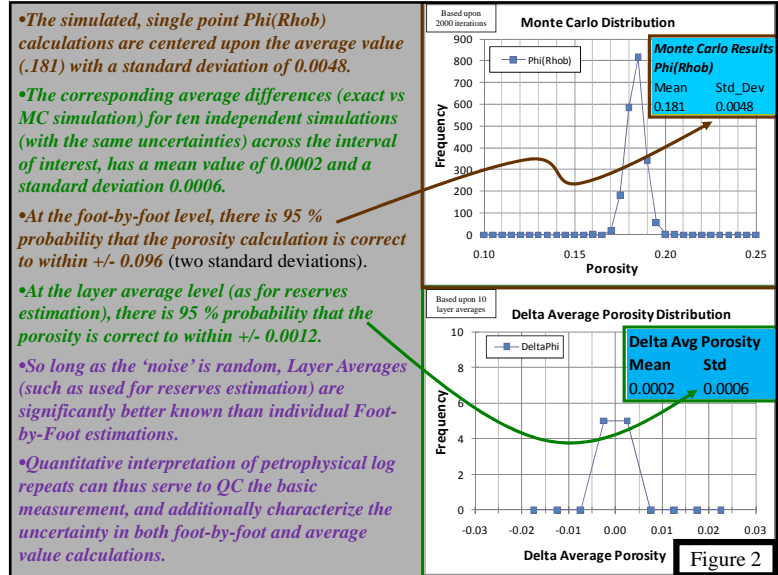
“Noise” tends to average out, and as a result layer averages may be much better known (and our reserves calculation more accurate) than the foot-by-foot deviations would suggest: Figure 2. In this example, **the layer**

average value exhibits a standard deviation of 0.0006, corresponding to a 95 % probability band of + / - 0.0012, as compared to + / - 0.0096 for the foot-by-foot situation.

What is happening here is **related to the Central Limit Theorem**, which states that as the sample size of a given population increases, the corresponding statistical description of that population is better known.

Cautions to be exercised in this use of the data include the following.

- The “noise” must be random and not a “bias”.
 - If a “bias” appears in the observed Main vs Repeat results, that is an important observation that requires additional action.
- In the case of a legacy database, or one which includes multiple service companies, we should be alert for possible variations in tool performance across time and vendor.
- Comparisons to core should recognize the different volumes of investigation, and evaluation of pad tools the possibility of variable orientations.
- When layer average maps are constructed, and contoured, these observed average difference distributions can be considered (is the bulls-eye a statistical possibility?).
- Most of our interpretations “assume” some kind of “model” (mathematical relation), and the potential for an inappropriate “model” should not be over-looked.
 - Interpretation models may be more appropriate in certain conditions than in others.
- In the quest for Model Improvement, we should recognize that it is quite possible that different input attributes have different impacts (more or less) on the final estimate, and that the Biggest Bang for the Buck should be determined for each locally specific set of conditions.



In any case **there is value in routinely establishing the simple statistical attributes of the observed Main vs Repeat pass log traces at both the foot-by-foot and layer average levels. The focus of uncertainty calculations is typically at the foot-by-foot level, and we may very well find that our layer averages are better known.** We will also then have available a quantitative logging tool QC reference.

The Basic Data and Interpretation

Uncertainty is present in most of the things we do, and as conscientious geoscientists we are always seeking to improve our results. **Since time and budget are limited, a technique which could identify which of the various inputs has the greatest impact on the ultimate estimate, and thereby focus our attention, would be an obvious starting point.**

There are two basic ways in which The Biggest Bang for the Buck can be identified: partial derivatives and statistical simulation. Although **the concepts are illustrated with petrophysical log calculations,** it is important to **remember that the techniques are equally valid for other actions: routine core analysis, directional survey bottom-hole placement, etc.** In actual fact, we got the idea for the petrophysical Excel Monte Carlo models used herein from a geological calculation of reservoir volumes that was posted on the LSU www site (<http://www.engr.lsu.edu/pttc/>).

There is often no single, constant answer to the question “which attribute is most important”: Figure 3. These results are based upon Chen and Fang’s differential analysis of Sw(Archie), and clearly reveal that in their **Base Case,** and **for Porosity < 20 pu, the “n” exponent is a relatively minor player.**

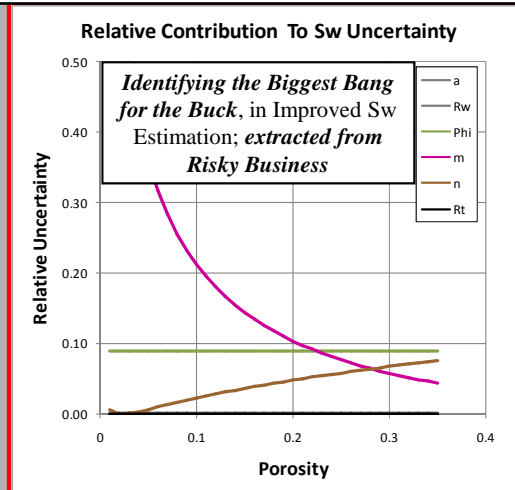
Figure 3

•**Right:** Relative impact on Sw(Archie) uncertainty of ‘m’ & ‘n’, across a range of porosity values, for a fixed Phi uncertainty.

•**Below:** Illustrative Best Estimate of each parameter, with corresponding individual uncertainty, and associated relative uncertainty on Sw(Archie), at a specific porosity.

Attribute Uncertainties Specified Individually			
Light Green Cells require User Specification			
Light Blue Cells are calculated results			
Attribute	Individual Uncertainty	Best Estimate	Relative Uncertainty On Sw(Archie)
a	0.0%	1.00	0.0000
Rw	4.4%	0.02	0.0019
Phi	15.0%	0.20	0.0900
m	10.0%	2.00	0.1036
n	5.0%	2.00	0.0480
Rt	1.0%	40.00	0.0001
Sw		11%	
Sw*n		1%	
Sw*n=0.367 is an inflection point			

After C. Chen and J. H. Fang. Sensitivity Analysis of the Parameters in Archie’s Water Saturation Equation. The Log Analyst. Sept – Oct 1986



•**The relative importance of ‘m’ and ‘n’ depend not only upon their specific uncertainty, but also upon the porosity of the interval in question; there is a link**

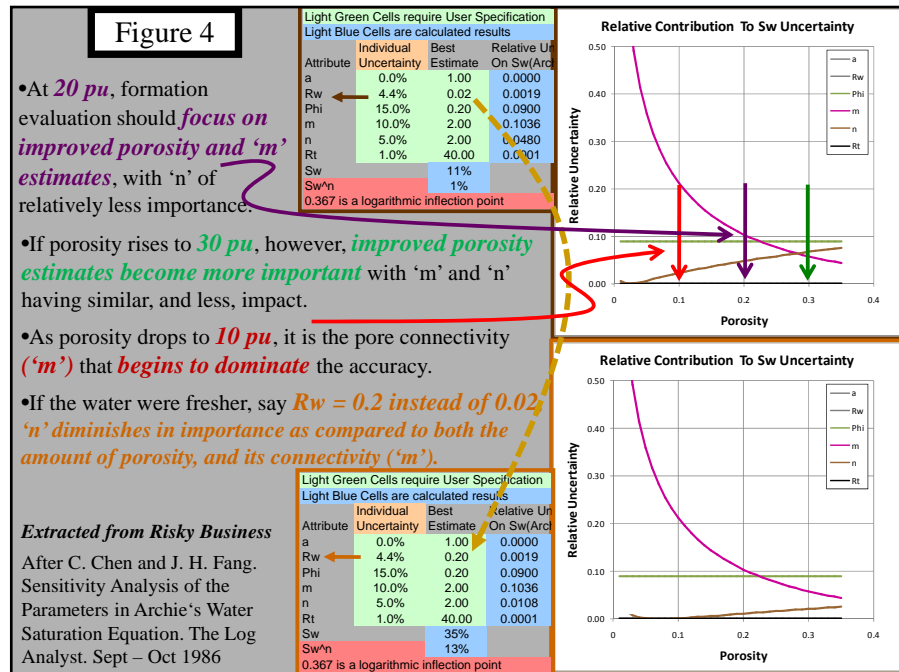
•Uncertainty resulting from ‘a’, R_w and R_t is below that of ϕ , ‘m’ & ‘n’ in this illustrative example

The lower the porosity, the greater the importance of the “m” exponent (pore connectivity), and so we find there can be a link between parameters.

Not only is *the importance of “m” tied to how much porosity is present*, but there is *also a dependence upon Rw*: Figure 4.

With a bit of reflection, we realize that physically as the water become fresher and Rw becomes larger, the issue of pore connectivity “m” may be yet more dominant than the wettability issue (exponent “n”).

Each situation must be addressed with locally specific attributes and uncertainties, and there may be a link between parameters.



The **second option for finding the Biggest Bang for the Buck is with Monte Carlo modeling**, which can be **implemented with routine Excel spreadsheet functions**. The Monte Carlo method randomly assigns values, according to user specified probability distributions, to each of the input parameters and then calculates the result. When the simulation is repeated a statistically significant number of times (results herein are based upon 2000 passes, which Excel handles without a problem), one is able to determine the likely outcome within any specific probability band, and to further identify which parameter is dominating the uncertainty (and hence where time and money is most efficiently directed for an improved result). **The output statistical distribution, corresponding to local conditions, can also be compared to actual Main vs Repeat Pass comparisons**, and thereby reveal if our model calculations are similar to empirical observations; a discrepancy would be reason for further investigation.

Construction of a physically representative Monte Carlo model will illustrate both the generic Monte Carlo concept, and provide a numerical model that can be further invoked to exhibit the differences between foot-by-foot and layer average uncertainties.

Phi(Rhob) is deduced from the various input attributes according to the following.

$$R_{hob} = R_{hof} * \Phi(R_{hob}) + R_{hog} * [1 - \Phi(R_{hob})]$$

Uncertainty is present in each of the three input quantities: Rhob, Rhof and Rhog.

Consider for the moment a calcite – dolomite mineral mix, for which the exact concentration is uncertain to 10 %. If the endpoint grain densities are taken as 2.71 gm/cc and 2.87 gm/cc, we calculate the following mixed results.

- Rhog(90 % calcite ⇔ 10 % dolomite) = 2.726 gm/cc
- Rhog(10 % calcite ⇔ 90 % dolomite) = 2.854 gm/cc

If the interval is thought to be 100 % calcite, the grain density could in fact be 2.726 gm/cc, or some 0.016 gm/cc larger than assumed. At the other end of the spectrum, if the interval is thought to be 100 % dolomite, the grain density could in fact be 2.854 gm/cc, or some 0.016 gm/cc lower than assumed. At the simplest level there is about 0.03 gm/cc uncertainty in Rhog. Recalling that +/- two standard deviations will encompass 95 % of the “noise”, we are prompted to take the standard deviation of Rhog to be $[0.03 \text{ gm/cc}] / 4 \sim 0.0075 \text{ gm/cc}$.

Fluid density Rhof is dependent upon the relative amounts of mud filtrate, connate water and hydrocarbon, in addition to temperature and pressure. For the purposes of illustration, let us take the mud filtrate to be 50 kppm and the connate water as 150 kppm (similar to many Middle East environments). At reservoir conditions the corresponding densities would be about 1.011 gm/cc and 1.085 gm/cc.

If the hydrocarbon density is taken as 0.700 gm/cc, the specification of the relative concentrations, and associated individual uncertainties, will characterize the average fluid density and its uncertainty.

Illustrative concentrations yield the following fluid densities.

- Rhof(80% MF, 0 % CW, 20 % Unflushed Hydrocarbon) = 0.949 gm/cc.
- Rhof(60% MF, 20 % CW, 20 % Unflushed Hydrocarbon) = 0.964 gm/cc.
- Rhof(40% MF, 40 % CW, 20 % Unflushed Hydrocarbon) = 0.978 gm/cc.
- Rhof(20% MF, 20 % CW, 20 % Unflushed Hydrocarbon) = 0.993 gm/cc.

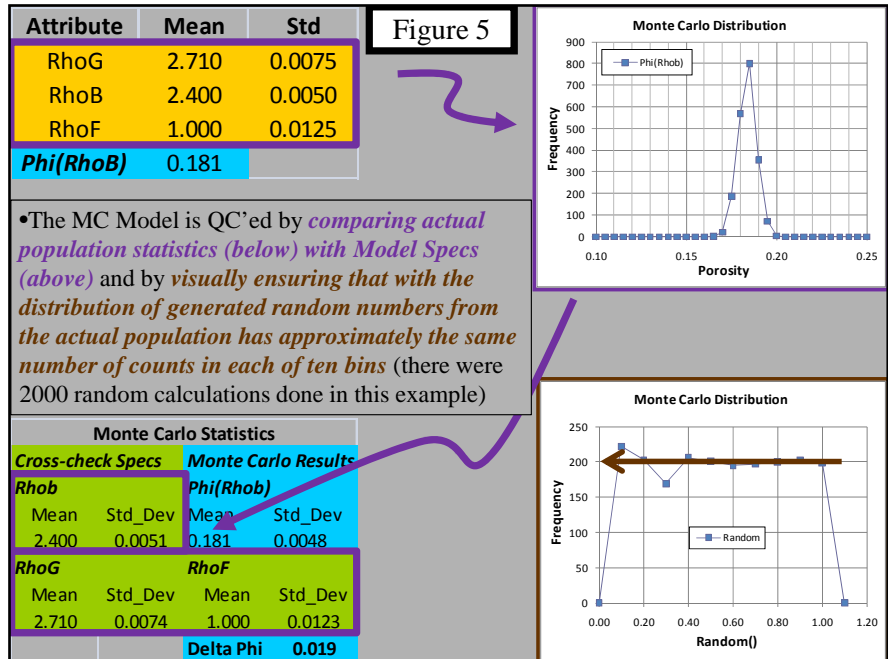
Mud filtrate invasion and displacement of connate water and hydrocarbon can be a fairly complicated and variable situation (David Allen, 2005), and the assignment of locally specific values deserves some careful consideration. For illustration purposes we have taken the Rhof uncertainty in these calculations to be 0.05 gm/cc. Two standard deviations high and low will encompass 95 % of the “noise”, so that the example standard deviation is taken as $[0.05 \text{ gm/cc}] / 4 = 0.0125 \text{ gm/cc}$.

Modern bulk density tools will typically repeat fairly good, and one public domain uncertainty value on a single Rhob measurement is + / - 0.01 gm/cc, corresponding to a standard deviation of $[0.02 \text{ gm/cc}] / 4 = 0.005 \text{ gm/cc}$.

With the above assumptions, we find that the largest standard deviation is that of Rhof, driven by the uncertainty associated with the invasion process. Because the pore volume is likely only one quarter the bulk volume (or less), however, the impact of Rhof on the ultimate porosity estimate is discounted relative to the uncertainty of Rhog (because grain volume is larger than fluid volume).

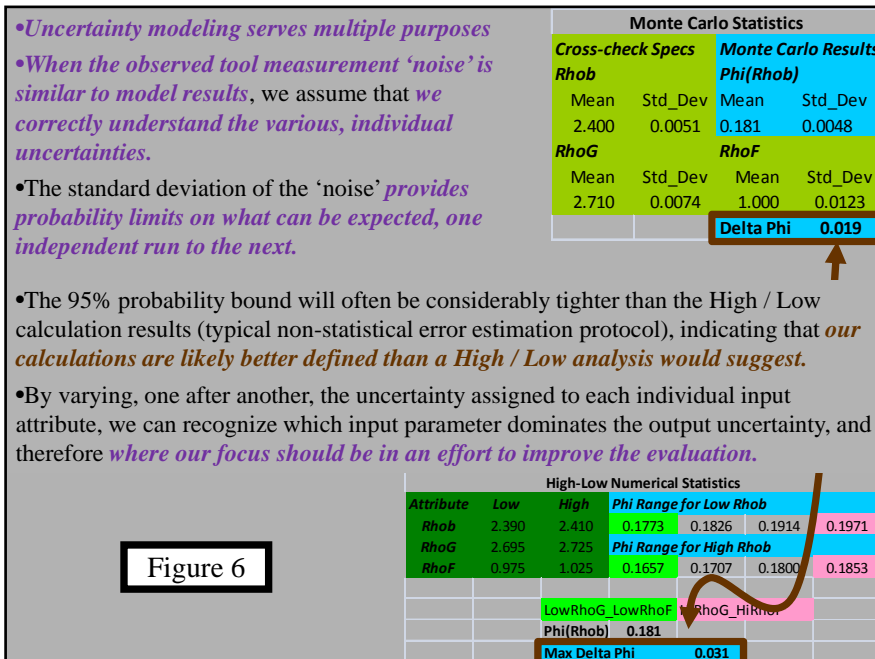
The Excel Monte Carlo simulation is executed based upon 2000 independent calculations: Figure 5. Two QC checks are implemented.

- The distribution of random numbers between 0 and 1, for each of the independent calculations, is histogrammed and we expect to find an equal number of events in each bin.
- Population statistics are calculated on the independent Monte Carlo simulations and compared to the independent model specifications. Agreement confirms that the MC population is representative.



In practice there could be an additional QC point; one would cross-check the Monte Carlo distribution (based upon locally appropriate specifications) against empirically observed Main – Repeat Pass measurements.

In examining the MC model results another advantage of simulation surfaces (beyond this discussion), where we notice that the 95 % result boundary is considerably narrower than is the (common) High – Low Deterministic Estimation: Figure 6.



This happens because it is unlikely (but not impossible) that the individual variations which yield the High (or Low) value will all occur simultaneously. **Our estimates are typically better than a High – Low calculation will indicate.**

We should **be aware that while Monte Carlo modeling normally assumes independence of the various attributes, that is not always the case.**

In a two mineral carbonate environment one link we might expect would be that the better porosity is associated with dolomite, rather than calcite, based upon the different molecular volumes.

In practice, however, there may be a correlation between core porosity ↔ core grain density, but with the better porosity associated with the higher calcite concentrations: Figure 7.

Jerry Lucia (2004) tells us

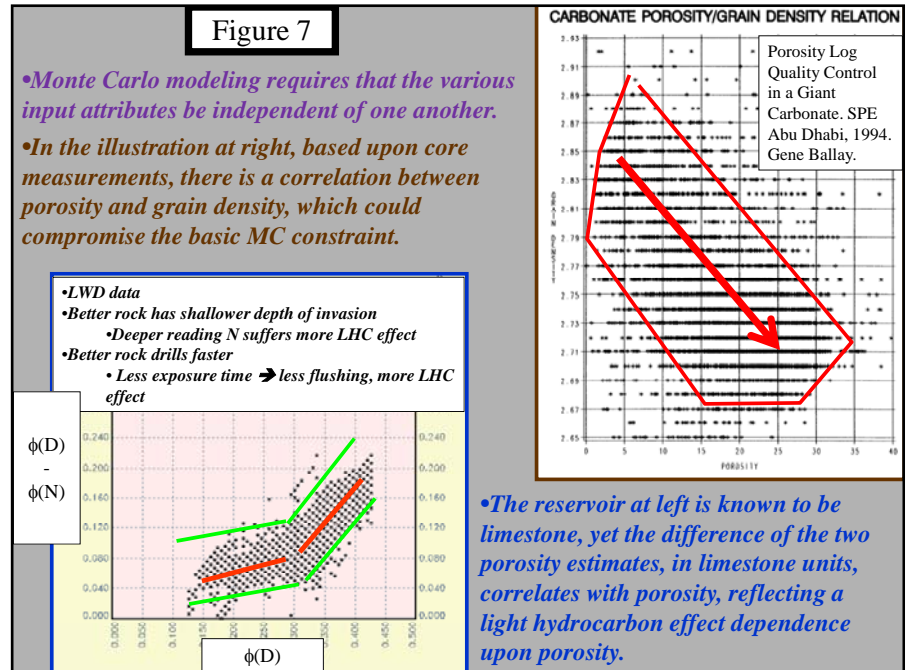
“The common claim that dolomitization creates 12% porosity is based on the mole-for-mole replacement equation. Over the past 50 years data have been collected demonstrating that porosity can be reduced by over dolomitization.”

Another example of a possible correlation is that of light hydrocarbon effects and porosity magnitude: Figure 7. The crossplot is of $\Phi(\text{Rho}) - \Phi(\text{Neutron})$ vs $\Phi(\text{Rho})$ in a limestone reservoir. As porosity increases so too does the difference in the Density and Neutron, reflecting the different depths of invasion (higher porosity typically invades less deeply) and the correspondingly different light hydrocarbon effects on the two tools, which have their individual depths of investigation. This graphic also illustrates why the GOC might be more obvious in higher porosities than in tighter rock.

Given that the interval is known to be limestone, we realize that the light hydrocarbon identification / correction algorithm will need to be variable, or else there will be an unaccounted for effect in the final estimate, that correlates with the porosity magnitude.

In light of carbonate pore system complexity and variation, we realize that one also needs to be alert for variations (and correlations) in the cementation exponent as discussed by Focke and Munn (1987).

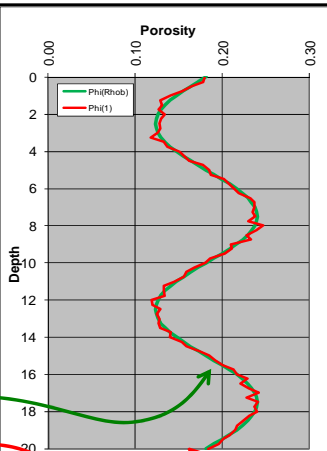
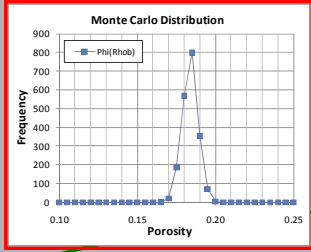
In summary, construction of the MC model and interpretation of the “noise” in empirically observed results is more than simply “plugging and chugging”. Which is why a good petrophysicist cannot be replaced by a computer.



Foot-by-foot versus Layer Averages

•The foot-by-foot uncertainty, versus layer average uncertainty, is illustrated by *superimposing the illustrative, realistic Phi(RhoB) distribution* upon a sine curve, where *the sine curve is taken as the 'exact' formation value* and *the illustrative 'real world' log measurement is the MC simulation.*

•Multiple MC realizations are generated and compared to characterize the 'uncertainty' associated with a single layer average.



	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
4	Phi(RhoB)	RhoG	RhoF	RhoB	Phi(t)		RhoG	RhoF	RhoB	Phi(t)		RhoG	RhoF	RhoB	Phi(t)
5	0.181287	2.709	1.007	2.392	0.187		2.696	1.003	2.408	0.170		2.713	0.989	2.399	0.182
6	0.172138	2.714	1.014	2.418	0.174		2.705	0.989	2.414	0.170		2.722	1.003	2.422	0.175
7	0.163215	2.705	0.998	2.429	0.162		2.709	0.994	2.426	0.165		2.708	1.011	2.426	0.166
8	0.154737	2.712	1.019	2.452	0.154		2.709	1.002	2.445	0.155		2.723	0.989	2.446	0.159
9	0.146913	2.699	1.005	2.460	0.141		2.700	1.037	2.461	0.143		2.719	1.009	2.459	0.152
10	0.139935	2.704	1.003	2.471	0.137		2.721	1.009	2.473	0.145		2.707	1.000	2.478	0.134
11	0.133976	2.711	0.993	2.481	0.134		2.715	0.995	2.482	0.138		2.717	0.978	2.488	0.132

Figure 8

The difference between individual incremental uncertainty and layer average differences can be illustrated with the preceding, realistic Phi(RhoB) distribution. A single ideal logging pass is represented with a sine curve, and the corresponding "noisy" logging pass with the MC distribution superimposed upon that sine curve: Figure 8.

The convergence of layer averages to the mean is characterized by the distribution of multiple independent layer average simulations: Figure 9.

So long as the Main ⇔ Repeat Pass differences are "noise" and not "bias", they will tend to average out across the repeat interval. In some instances the Main Pass will be high to the Repeat Pass, and in others low, as seen in the Delta column of Figure 9.

Even though the standard deviation of the "noise" in the Main Pass is 0.0048, the layer average difference in Main ⇔ Repeat never exceeds 0.001 and is often considerably less.

If we don't have the information or time to construct a locally appropriate MC model, we could rely upon the empirically determined Main ⇔ Repeat statistical attributes. Once the locally appropriate spreadsheet is set up, whether by empirical observation or MC simulation, hitting the F9 key will cause Excel to recalculate and allow us to visually page through the various possibilities.

	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
4	Phi(RhoB)	RhoG	RhoF	RhoB	Phi(t)		RhoG	RhoF	RhoB	Phi(t)		RhoG	RhoF	RhoB	Phi(t)
5	0.181287	2.709	1.007	2.392	0.187		2.696	1.003	2.408	0.170		2.713	0.989	2.399	0.182
6	0.172138	2.714	1.014	2.418	0.174		2.705	0.989	2.414	0.170		2.722	1.003	2.422	0.175
7	0.163215	2.705	0.998	2.429	0.162		2.709	0.994	2.426	0.165		2.708	1.011	2.426	0.166
8	0.154737	2.712	1.019	2.452	0.154		2.709	1.002	2.445	0.155		2.723	0.989	2.446	0.159
9	0.146913	2.699	1.005	2.460	0.141		2.700	1.037	2.461	0.143		2.719	1.009	2.459	0.152
10	0.139935	2.704	1.003	2.471	0.137		2.721	1.009	2.473	0.145		2.707	1.000	2.478	0.134
11	0.133976	2.711	0.993	2.481	0.134		2.715	0.995	2.482	0.138		2.717	0.978	2.488	0.132

Average Values	Delta	Delta Avg Porosity Mean	Delta Avg Porosity Std
Exact	0.1813	-0.0001	0.0007
Phi(1)	0.1817	-0.0004	
Phi(2)	0.1809	-0.0004	
Phi(3)	0.1817	0.0004	
Phi(4)	0.1801	-0.0012	
Phi(5)	0.1806	-0.0006	
Phi(6)	0.1805	-0.0008	
Phi(7)	0.1818	0.0005	
Phi(8)	0.1814	0.0001	
Phi(9)	0.1807	-0.0006	
Phi(10)	0.1822	0.0009	

•Each time the spreadsheet is opened the calculations are re-seeded and the estimates will change.

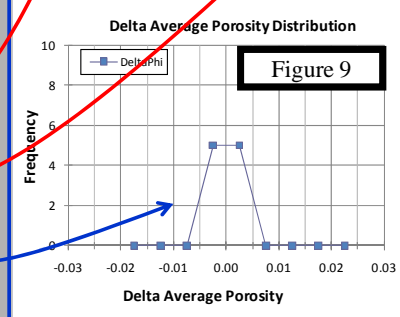


Figure 9

•Multiple foot-by-foot MC realizations are generated and compared to characterize the 'uncertainty' associated with a single layer average.
 •The standard deviation of the various layer average estimates is considerably smaller than that of the foot-by-foot simulation, indicating that layer averages are better known.

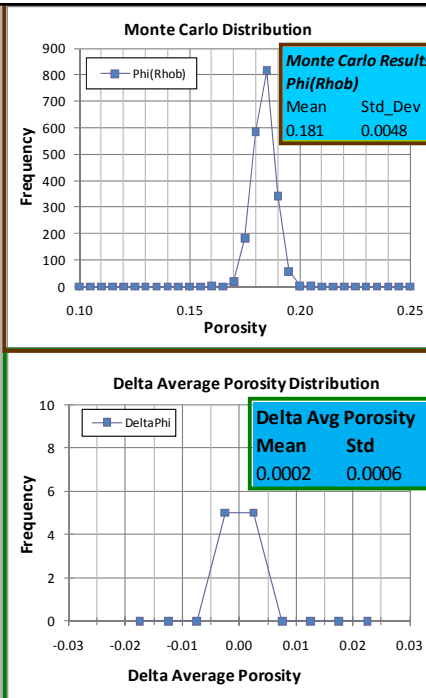
•At the foot-by-foot level, there is 95 % probability that the porosity calculation is correct to within +/- 0.096 (+/- 2 * 0.0048).

•At the layer average level (as for reserves estimation), there is 95 % probability that the porosity is correct to within +/- 0.0012 (+/- 2 * 0.0006).

•So long as the 'noise' is random, Layer Averages (such as used for reserves estimation) are significantly better known than Foot-by-Foot estimations.

•Quantitative interpretation of actual petrophysical log repeats can not only serve to QC the basic measurement, but to also characterize the uncertainty in both foot-by-foot and average value calculations.

Figure 10



In the case at hand, there is 95 % probability that the porosity calculation is correct to within +/- 0.096 (+/- 2 * 0.0048) at the foot-by-foot level: Figure 10.

At the layer average level (as for reserves estimation), there is 95 % probability that the porosity is correct to within +/- 0.0012 (+/- 2 * 0.0006).

So long as the 'noise' is random, Layer Averages (such as used for reserves estimation) are

significantly better known than Foot-by-Foot estimations. Quantitative interpretation of actual petrophysical log repeats can not only serve to QC the basic measurement, but to also characterize the uncertainty in both foot-by-foot and average value calculations.

The reason for the convergence of the layer averages is perhaps more apparent if the sine curve is flattened: Figure 11. The spreadsheet contains a Hi_Low setting that controls the range

of the sine curve; when Hi_Low is set to 0.0, representing a bed of uniform porosity, the "noise" is centered upon that single porosity value, high and low, in a random fashion. There is an analogy here to the Central Limit Theorem, which tells us that the greater the number of samples taken from a specific population, the closer to the true value will be the characteristics of the sample population.

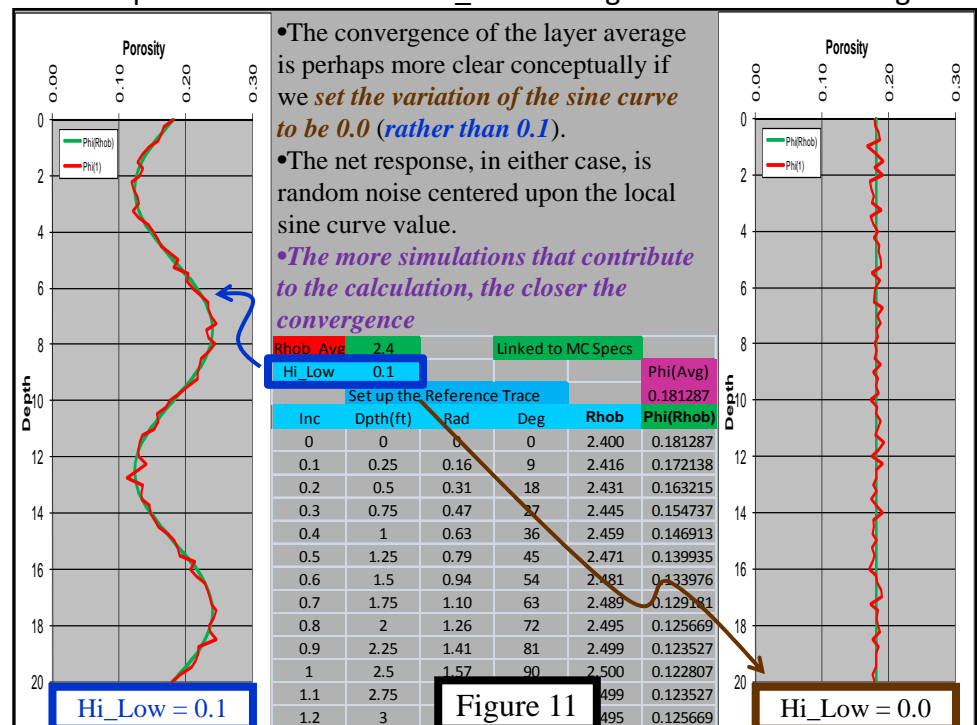
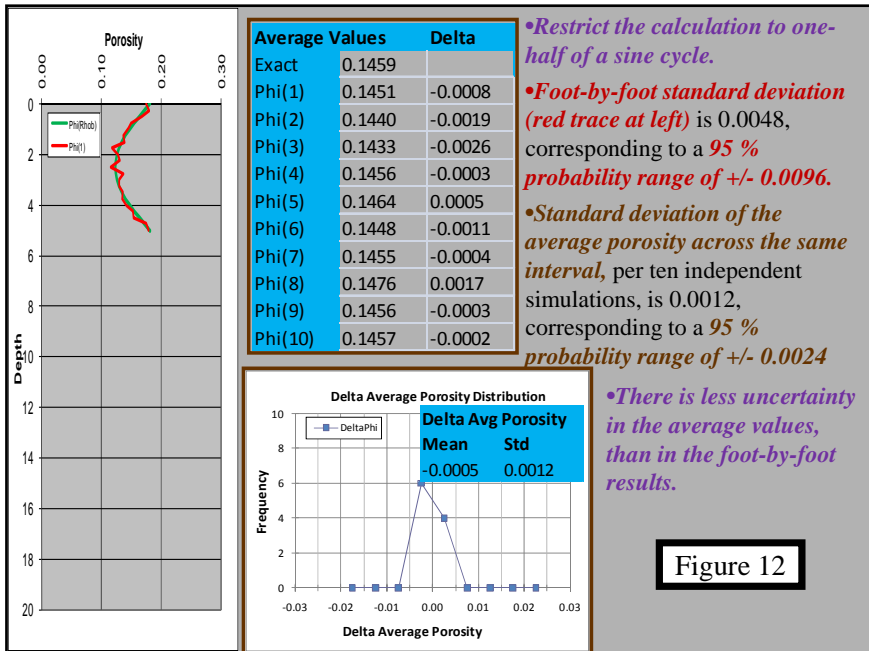


Figure 11



In the Oil Field we seldom have intervals of constant porosity, or complete sine curve cycles, but so long as the “noise” is random, there will be high and low estimates which surround the individual foot-by-foot values, and the layer average will smooth out the “noise”:

Figure 12.

Caution

In many cases we won't have the time or information necessary to model a tool response, and will then revert to use of the empirically observed Main ↔ Repeat Pass traces to characterize the “noise” present in a specific measurement. In so doing, **we must make sure that we have made every effort to recognize and eliminate** (or account for) “bias”.

Bias can arise for several reasons, with poor depth control being one of the first issues that comes to mind. When the Main and Repeat Passes are put on depth, prior to the statistical characterization, we

need to consider supplemental information as appropriate.

Figure 13 compares the GR that was acquired while drilling, with the drill string stressed and torqued, against the GR from the pipe-conveyed logs (stress and torque relaxed). **Drilling conditions have “wrapped up” about three feet of**

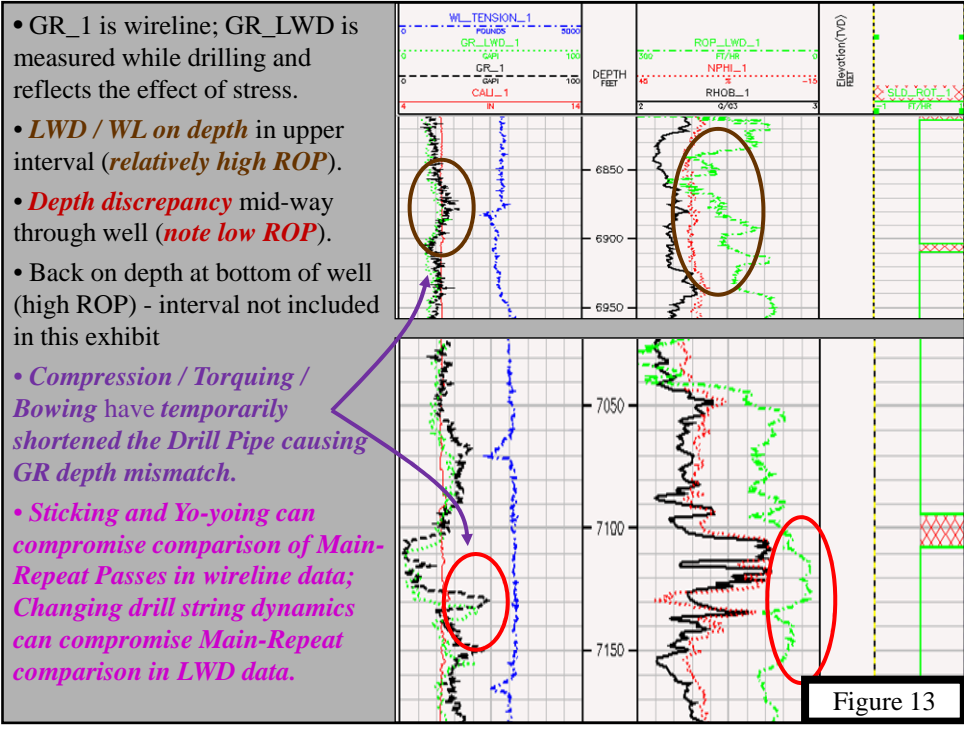
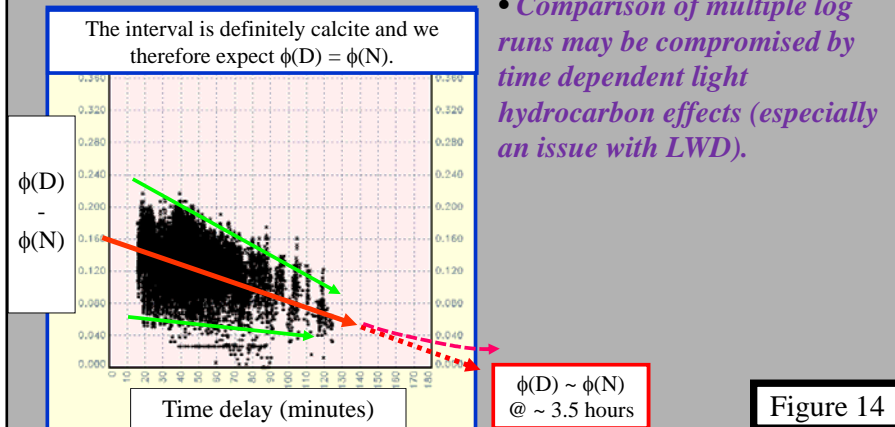


Figure 13

Invasion, Light Hydrocarbon Effect and Lag Time

- **Density - neutron separation versus formation penetration time lag.**
- **Light hydrocarbon effect (density - neutron separation) and logging delay (time after bit penetration) have some correlation.**
- $\phi(D)$ & $\phi(N)$ approach one another with increased time (invasion).



the pipe, such that the actual depth along the wellbore with the LWD measurement is less than the length of pipe that has been run in the hole. **Above and below the slow drilling (high stress) interval, LWD and pipe-conveyed measurements are on depth.**

Invasion is another potential “bias” effect. The reservoir in Figure 14 is a light oil limestone, with porosity from the density and neutron contrasted as a

function of lag time. The measurements are LWD and the combination of tool dimensions and drilling penetration rate allow one to calculate the time lapse between penetration of the formation by the bit (first exposure to mud filtrate) and the measurements.

Since the interval is known to be limestone, we expect $\Phi(\text{Density}) \sim \Phi(\text{Neutron})$, and indeed we find the extrapolation (in time) of the porosity difference to trend in exactly that direction. In practice, however, much of **the data is light hydrocarbon affected in an amount which depends upon the lag time.**

In some cases we may not be able to completely eliminate the possibility of bias, but that does not mean we should discount the utility of quantitative comparison of Main ⇔ Repeat Passes. Rather we should carry along with the evaluation, comments and attributes that are thought to be factors. As the data base grows we then expect a convergence of results; if this doesn't happen then we have over-looked something.

3-D Uncertainties

As a petrophysicist, our focus is typically on the foot-by-foot and layer average results. Downstream of us, though, someone is likely going to initialize a simulator, or calculate reserves, and at this point the layer averages and associated statistical properties will be of interest.

As discussed above, the layer average “noise” may not be the same for each of the various average estimates. The tool generation, tool suite, service company, etc may be different with each having its own specific characteristics.

If the maps are manually drawn, or computer drawn and then manually edited, the geologist may reference the individual layer average statistics in determining how closely a particular contour can be expected to agree with a single well value.

Alternatively, many practitioners will address the question with geo-statistical package, where the uncertainty issues are a part of the extrapolation / in-filling.

An over-view of the options is discussed by Raghu Ramamoorthy at www.SPWLA-AbuDhabi.com, in “Well Data Uncertainty and its role in the Static Model”.

Summary

In most evaluations, ***the Log Repeat seldom receives any detailed attention*** beyond possibly a simple comment such as ‘repeat looks reasonable’. Were we to ***take the time to digitally load the Repeat and compare it to the Main Pass in both the foot-by-foot and average value sense***, we would be able to not only ***better QC each logging run individually***, but ***also estimate the uncertainty present in the layer average values***.

“Noise” tends to average out, and as a result layer averages may be much better known (and our reserves calculation more accurate) ***than the foot-by-foot deviations would suggest***.

The effect has been illustrated by construction of a physically realistic Monte Carlo Phi(Rhob) model, followed by characterization of the simple statistics associated with both foot-by-foot and layer average estimates.

Cautions to be exercised include the following.

- The “noise” must be random and not a “bias”.
- In the case of a legacy database, or one which includes multiple service companies, we should be alert for possible variations in tool performance across time and vendor.
- Most of our interpretations “assume” some kind of “model” (mathematical relation), and the potential for an inappropriate “model” should not be over-looked.
- In the quest for Model Improvement, we should recognize that it is quite possible that different input attributes have different impacts (more or less) on the final estimate, and that the Biggest Bang for the Buck should be determined for each locally specific set of conditions.

In any case ***there is value in routinely establishing the simple statistical attributes of the observed Main vs Repeat pass log traces at both the foot-by-foot and layer average levels. The focus of uncertainty calculations is typically at the foot-by-foot level, and we may very well find that our layer averages are better known.***

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Biography

R. E. (Gene) Ballay's *35 years in petrophysics* include *research and operations* assignments in Houston (Shell Research), Texas; Anchorage (ARCO), Alaska; Dallas (Arco Research), Texas; Jakarta (Huffco), Indonesia; Bakersfield (ARCO), California; and Dhahran, Saudi Arabia. His carbonate experience ranges from individual Niagaran reefs in Michigan to the Lisburne in Alaska to Ghawar, Saudi Arabia (the largest oilfield in the world).



He holds a *PhD* in *Theoretical Physics* with *double minors in Electrical Engineering & Mathematics*, has *taught physics in two universities*, *mentored Nationals* in Indonesia and Saudi Arabia, published *numerous technical articles* and been designated *co-inventor on both American and European patents*.

At retirement from the Saudi Arabian Oil Company he was the senior technical petrophysicist in the Reservoir Description Division and had represented petrophysics in three multi-discipline teams bringing on-line three (one clastic, two carbonate) multi-billion barrel increments. Subsequent to retirement from Saudi Aramco he established Robert E Ballay LLC, which provides physics - petrophysics training & consulting.

He served in the U.S. Army as a Microwave Repairman and in the U.S. Navy as an Electronics Technician; he is a USPA Parachutist, a PADI Nitrox certified Dive Master and a Life Member of Disabled American Veterans.